Measurement of the $K^-p\rightarrow\Sigma^0\pi^0$ reaction between 514 and 750 MeV/c

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The reaction $K^- p \rightarrow \Sigma^0\pi^0 \rightarrow$ neutrals was studied with the Crystal Ball detector at the BNL Alternating Gradient Synchrotron for beam momenta between 514 and 750 MeV/c. The photons from $\Sigma^0$ and $\pi^0$ decays were detected, as well as the neutron in a small fraction of the events. The differential cross section and the polarization of the $\Sigma^0$ are reported at eight momenta and nearly the full angular range. Total cross sections are derived from Legendre polynomial fits to the data. The measurements presented are of considerably higher precision than previous results.


I. INTRODUCTION

A series of $K^- p \rightarrow$ neutrals reactions was studied at eight beam momenta between 514 and 750 MeV/c with the Crystal Ball detector [1–4], formerly used at the Stanford Linear Accelerator Center (SLAC) and at DESY. The experiment was performed at the BNL Alternating Gradient Synchrotron (AGS). This paper specifically describes, from among the several reactions detected, measurements for

$$K^- p \rightarrow \Sigma^0\pi^0, \quad (1)$$

where

$$\Sigma^0 \rightarrow \Lambda\gamma \rightarrow (n\pi^0)\gamma.$$
A. Beam line and beam counters

Primary protons were extracted from the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory into the C line. Some of these protons struck a 0.51 × 0.76 × 9 cm³ (w × h × t) platinum target, producing secondary particles at an angle ∼5° into the C6 beam line. The primary-proton beam intensity was approximately 5 × 10¹² per spill every 5.1 s, and the spill length was 2.8 s.

The secondary-particle beam was focused with quadrupole magnets and traversed dipole magnets and electrostatic separators; see Fig. 2. The secondary particles passed through a mass-selection slit after the second separator, enhancing the fraction of kaons in the beam to approximately 6–10%. The slit was formed from Hevimet (an alloy of tungsten with small amounts of copper and nickel), of thickness 10 cm along the beam, and the horizontal gap was open 4–9 mm during the kaon-beam runs described here.

The scintillation counter hodoscope, S₁, was located immediately downstream of the mass slit and approximately 681 cm upstream of the liquid-hydrogen target. This hodoscope consisted of eight identical vertical bars, each 1.8 cm wide. Most of the useful beam particles traversed the central five or six counters in the hodoscope, depending on the beam momentum. For a small fraction (∼5%) of the data, corresponding to the earliest measurements, this hodoscope was replaced with various background reactions and beam-particle decays. For some events, neutrons were detected in the CB.

For each event, the pulse heights in each NaI counter of the CB and for many of the plastic scintillation counters were recorded. Time information was also measured for sets of nine or fewer NaI counters and for individual photomultipliers on the scintillation counters. In addition, some of the liquid-hydrogen target and beam-line magnet operating parameters were recorded. Many histograms were available to monitor the data quality on-line. All of these data were recorded on disk, copied to magnetic tapes, and analyzed off-line.

Additional details are presented in the following sections and in a number of publications [4,31] and theses [32–35].
by a single scintillation counter, with the same area as the hodoscope, viewed by two photomultiplier tubes.

A momentum analysis of the beam particles was performed with the second dipole magnet, $B_2$ (see Fig. 2), and a set of four multiwire drift chambers, $D_1$–$D_4$. Chamber $D_1$ was located upstream and the other three chambers downstream of the $B_2$ magnet. Each chamber consisted of two wire planes to measure the horizontal ($x$) beam position. In addition, for all chambers except $D_1$, there were two wire planes to measure the vertical ($y$) beam position. The spacing between the sense wires was 5.08 mm, and the active areas were $122 \times 175$ mm$^2$ for all planes except $D_1$, which was $122 \times 50$ mm$^2$; all planes had 24 sense wires. The mean beam momentum was determined from measurements of the $B_2$ central magnetic field as a function of current, from the kinematics of some $\pi^-p$ reactions with the CB, and from past experiments in the C6 beam line. In addition to measuring the particle momentum, the chamber information was extrapolated to give the beam–particle position at the liquid-hydrogen target. Table I gives the mean beam momentum, momentum spread $\Delta p/p$ in FWHM (full width at half maximum), and spot size in FWHM at the target for each of the eight beam momenta used in this experiment. It is estimated that the mean beam momentum is known to approximately $\pm 3$ MeV/c.

The beam particles were also detected in scintillation counters $S_2$ and $S_T$, where counter $S_2$ was located immediately upstream of chamber $D_2$, 320 cm upstream of the liquid-hydrogen target, and $S_T$ was positioned closer to the Crystal Ball, 162 cm upstream of the target. The $S_2$ and $S_T$ counter dimensions were $12.7 \times 7.6 \times 0.3$ cm$^3$ and $7.6 \times 5.2 \times 0.6$ cm$^3$, respectively. Two photomultipliers viewed $S_T$ for improved timing resolution, but only one was used on $S_2$. The time of flight (TOF) of particles between $S_1$ and $S_T$ was used to select kaons in the beam and to estimate backgrounds from pions; see Fig. 3.

The beam intensity was monitored using the electronic coincidence, $K_{\text{BEAM}} = S_1 \cdot S_2 \cdot S_T$. In this signal, $S_T$ was formed from the coincidence of the two signals from the photomultipliers viewing this counter. The $S_1$ signal was a logical OR of the eight hodoscope counters, vetoed by the simultaneous presence of two or more hodoscope signals, allowing only one element of the hodoscope to fire within a 6-ns time window. The relative time delay between the $S_1$ and $S_T$ signals was set such that most of the pions in the beam were eliminated. The average beam flux during the spill at each momentum is given in Table I. Rates in the scintillation counters extended up to several MHz during the spill, primarily from the pions in the beam.

![FIG. 2. (Color online) C6 beam line, including production target, dipole and quadrupole magnets, electrostatic separators, and the last two magnets in the primary-proton line before the target.](image1)

![FIG. 3. TDC spectra for one $S_1$ hodoscope counter and one run at 514 and 750 MeV/c. The large peaks are from kaons, and the time per channel is 0.05 ns. The $S_T$ signal provided the stop timing, so pions would occur to the left of the kaon peak.](image2)
A set of four scintillation counters (HV) was used to veto particles in the halo of the beam. Each scintillator had dimensions $15.2 \times 29.2 \times 0.5$ cm$^3$, and they were located left, right, above, and below the nominal beam position, forming a rectangular hole. The size of the hole could be adjusted, and was set to $6.05 \times 4.60$ cm$^2$ for most of the kaon-beam runs. A variety of shielding and collimation was also present. A cylindrical steel collimator with an inner diameter of 10.2 cm and an outer diameter of 18.4 cm in a 61-cm-thick shielding wall was located after the last beam-line magnet, $Q_7$. A second collimator was formed from lead bricks in another 61-cm-thick concrete shielding wall, which was situated after chamber $D_4$. It was expected that this shielding significantly reduced backgrounds in the CB. However, backgrounds were observed from the primary proton beam, even when the C6 beam-line magnets were turned off. The simulation of the effects of some such backgrounds is described below in Sec. III E.

B. Liquid-hydrogen target

The liquid-hydrogen target was located at the approximate center of the Crystal Ball. It was a 10.2-cm-diameter and 10.6-cm-long cylinder with rounded ends. (The maximum liquid-hydrogen length, along the beam axis, was 10.6 cm.) The target flask consisted of $0.18$ mm Mylar walls, surrounded by layers of thin, aluminized-Mylar “super-insulation.” The target was located within a vacuum pipe of length 244 cm, also centered on the CB, with a vacuum box attached to the downstream end that connected to the hydrogen refrigerator. The aluminum vacuum pipe outer diameter was 15.2 cm, with a wall thickness of 0.21 cm. This pipe was lined with a 0.25-mm-thick Mylar sleeve for 40 cm, centered on the target. Stainless-steel fill and exhaust lines ran horizontally to the downstream vacuum box and then vertically to the refrigerator. Windows on the ends of the vacuum pipe and box for the beam, including those for safety as well as vacuum, contained approximately 0.6 mm of Mylar and Kapton at each end.

The target was operated at a temperature of about 20 K, and the number density was $4.23 \times 10^{-5}$/cm mb. The temperature was recorded every 5 s and written to the data files. Both full- and empty-target measurements were made at each beam momentum to estimate the effects of nonhydrogen backgrounds in the data. An empty-target run was typically performed after several full-target runs in a repeating pattern, with about a half hour to empty or fill the target.

C. The Crystal Ball

The Crystal Ball detector has been previously described in Refs. [1–4]. It consisted of 672 NaI crystals in the shape of truncated, triangular pyramids, each viewed from the outside with a single photomultiplier tube. The crystals were all nearly the same shape, with a length of 40.6 cm (15.7 radiation lengths), and were optically isolated from neighboring crystals with white paper and aluminum foil. If complete, the CB would have consisted of 720 crystals forming an icosahedron with an inner diameter of 50.8 cm; however, two sets of 24 crystals were absent on opposite sides to allow the beam to enter and exit. These regions with the missing crystals were termed “tunnels.” The two sets of 30 NaI counters adjacent to the openings are called the “guard” crystals; the energy resolution for photons striking these crystals is poorer than for the other crystals due to loss of shower energy into the tunnel. Each of the 20 triangular faces of the icosahedron was of the same size and were subdivided into four “minor triangles” of nine counters each.

Since NaI is hygroscopic, the crystals were located in two hermetically sealed, evacuated hemispheres. The lower hemisphere was fixed, but the upper hemisphere could be raised on three motorized jacks to gain access to the vacuum pipe and veto counters located inside the CB. In addition, the detector was situated inside a temperature-controlled (20 ± 1° C) dry room, where the dew point was −42° C.

The CB photomultiplier tubes were connected to one of four high-voltage power supplies and thus had common input voltages. Each photomultiplier tube could have the internal voltage in its base adjusted with a potentiometer to match approximately the output-signal gains. This procedure was done using a $^{137}$Cs source that was placed in the center of the CB when the hydrogen target and vacuum pipe were removed, preceding the data-taking runs. No further adjustments were made to the photomultiplier tubes or bases during the run. The final NaI counter gains were determined from pion-beam data, as described in Sec. III A.

D. Charged-particle detectors

Several plastic scintillation counters were used as veto counters so that the recorded data corresponded to all-neutral final states originating from interactions in the liquid-hydrogen target. These scintillators consisted of a pair of beam veto counters (BV, BVS) located downstream of the CB, a set of four counters forming a veto barrel (VB) immediately outside the beam pipe and inside the array of NaI counters, and a set of four scintillators (WV) inserted into the upstream tunnel of the CB; see Fig. 1.

The primary purpose of the beam veto counters was to reject events when the beam particle did not interact in the target or suffered a small-angle scattering. The dimensions of these counters were $15.2 \times 15.2 \times 0.6$ cm$^3$ for BVS and $91.4 \times 91.4 \times 1.0$ cm$^3$ for BV. They were located 211 cm (BVS) and 221 cm (BV) downstream of the target and were viewed by one and two photomultiplier tubes, respectively. The BV and BVS counters were not used in the trigger, but the integral of the pulse from the corresponding photomultiplier tubes was recorded for each event.

The veto barrel consisted of four identical counters of length 1.20 m and thickness 5.0 mm, and formed a cylindrical shell with an average diameter of 16.1 cm. The veto barrel counters were centered as a set on the liquid-hydrogen target and were mounted to the vacuum pipe. Each scintillator was viewed by two photomultiplier tubes, one at each end. The scintillator thickness was kept to a minimum to reduce photon conversions before reaching the NaI crystals.

As a result of the small scintillator thickness and long counter length, light-attenuation effects in the VB counters
were significant. In the data event trigger, a coincidence between signals from the two photomultipliers on each counter was required to reduce a random veto due to noise. Thus, the efficiency for detecting minimum-ionizing charged particles varied with position. Cosmic rays were used to map the variation of detection efficiency with position in the veto barrel counters, which was important for estimating acceptance and backgrounds; see Sec. III C. Simulations were also important for estimating the conversion of γ rays in the veto barrel counters or beam pipe and their impact on the event acceptance.

The WV counters were installed to eliminate some event triggers caused by beam-kaon decays in flight upstream of the CB. These counters were formed of a trapezoid-shaped, plastic scintillator of thickness 0.5 cm, parallel sides of dimensions 28.4 and 38.6 cm, and height 60.2 cm, and their light output was collected using wavelength-shifting optical bars. These four counters were placed nearly symmetrically around the nominal beam, tilted to yield an opening 28 cm × 28 cm closer to the target and an opening 39 cm × 39 cm upstream. The center of these counters was located 89 cm upstream of the target center. The OR of signals from these counters was used to veto the event trigger, reducing the rate by ∼7–10%, but the efficiency as a function of position was not mapped in detail.

E. Trigger and readout electronics

The analog signal from each NaI counter was sent to an integrate-and-hold module [1] with both low- and high-gain outputs. The signal from each output went into a separate channel of a LeCroy FERA [36] analog-to-digital converter (ADC). The low-energy range corresponded approximately to 0–50 MeV, and the high-energy range to 0–1000 MeV. The integration time for the ADC was 350 ns. The nine photomultiplier tube signals from a CB minor triangle went into a single integrate-and-hold module, which then also summed the nine pulse heights. The summed outputs from all counters, excluding the guard crystals, were added to give the signal, ΣCB. In addition, the summed signals from each minor triangle were discriminated and their times of arrival recorded with multihit time-to-digital converters (TDCs).

Signals from the S1, S2, S3, BV, and BV5 counters were also recorded in both TDCs and ADCs, to allow further analysis and tests of the data. The HV and WV counters had only ADC information recorded, and the VB counters had only ADC data. The S1 hodoscope counters had separate TDCs but a common ADC.

Several different event triggers were formed and used during data collection. A KBEAM event trigger was a coincidence of signals from one and only one S1 hodoscope counter, S2, and coincident signals from both photomultiplier tubes viewing S3. The coincidence of the two S3 photomultiplier tube signals defined the timing for this trigger. Tight conditions on the time difference (S1 − S3) rejected most pions, yielding kaon fractions in KBEAM from 79–94%. The KBEAM-trigger events were used to estimate corrections for beam decays, the effective target length, and beam contamination from pions, muons, or electrons; see Sec. III B. These triggers were only sampled during normal data-taking runs.

The main event trigger, CB-NEUTRAL, was a coincidence of KBEAM and the discriminated ΣCB signal, in anticoincidence with the electronic OR of the four VB, four HV, and four WV counters. The threshold for ΣCB varied with beam momentum and ranged from ∼200 to ∼350 MeV, excluding the energy deposited in the guard crystals. Again, the S3 counter defined the timing of the CB-NEUTRAL trigger and therefore the ADC gates and the TDC start (or stop) timing.

The third event trigger was a PULSER trigger that occurred only during the time between beam spills and whose rate was approximately once per second. This trigger allowed a determination of the pedestal values for all ADCs, including all NaI and plastic scintillation counters. (Pedestals measured in the NaI counters with the beam present were found to vary significantly with beam intensity.)

The number of usable kaons in the beam was monitored with a scaler, GOOD-BEAM. This GOOD-BEAM scaler counted the number of KBEAM signals in anticoincidence with the electronic OR of the four HV counter signals and the signal indicating that the computer was not busy reading an event. Typical experimental live times were between 75% and 90%. Various corrections to the GOOD-BEAM scaler were required, as described in Sec. III B.

The data were collected and analyzed online with the CODA software [37]. In addition to the ADCs and TDCs for the CB and plastic scintillation counters and wire chamber information, scalers were read after every spill to record rates or coincidences from various counters.

III. DATA ANALYSIS

A. Calibration of the Crystal Ball counter gains

Before taking data using the kaon beam, the high voltage of each NaI counter was adjusted to give roughly equal gains using a 137Cs radioactive source placed near the center of the CB. The low-energy range ADC for each counter was used to set the voltages, and the resulting gains were initially used on-line.

Final gains were obtained from about 50 000 pion charge-exchange events (π−p → π0n → γγn), collected shortly after the kaon-beam runs. Two- and three-cluster events were analyzed to reconstruct the π0 and π0n, respectively (see Sec. III D). The energy clusters from the two γ’s were required to be well separated, by at least 30°, to avoid summing energy from two showers in a single counter. After correcting for the ADC pedestals, the total energy was summed for each cluster, and the invariant mass IMγγ was computed using the cluster centroid locations. Next, for the entire charge-exchange data sample, the 2 × 672 gains for the two ADC ranges and 672 NaI counters were adjusted to minimize the width of the IMγγ peak while constraining the mean to be the π0 mass. The gains were adjusted for approximately 30 counters at a time, using those events with clusters contained in them, and this procedure was then iterated more than ten times for the entire sample. The results, after deriving the final gains for all NaI counters, are shown in Fig. 4. Gains were also
derived from $\pi^- p \rightarrow \pi^0 \eta$ events before the kaon runs, and little change was found compared with the adopted values. In addition, the final gains were used to reconstruct $\eta$'s from the $\pi^- p \rightarrow \eta n \rightarrow \gamma\gamma n$ reactions, giving a mass of IM$_{\gamma\gamma} = 548.8$ MeV/c$^2$ and width of $\pm24.5$ MeV/c$^2$. This gain determination procedure was verified during a different run period using stopped kaon decays, $K^+ \rightarrow \mu^+ \nu_\mu$, with the muon detected in the CB; see Ref. [4].

The accuracy of this gain determination was estimated from a simulation of the $\pi^- p \rightarrow \pi^0 \eta$ reaction using $\pi^0 \rightarrow \gamma\gamma$ events in a similar iterative procedure as described above. Starting values for the gains in this iterative process were chosen randomly and could be different from the input gains by up to 40% (FWHM). A comparison of the derived gains, after the iterations had converged, to the input gains indicated that the final gains were determined to within $\pm6\%$ for the low-energy range ADC channels and within $\pm\%$ for the high-energy range ADC channels. It is expected that the high-energy ADC channel gains are more accurately determined because IM$_{\gamma\gamma}$ is more sensitive to the gain of counters with higher energies in clusters. An attempt was made to constrain the ratio of gains for each counter using measured energies of $\sim20$–$50$ MeV, where useful values were recorded in both ADCs. However, this did not significantly improve the width of the IM$_{\gamma\gamma} = M_\pi^*$ peak or the centroids of the $M_\pi^0$ or $M_\eta$ peaks.

For the calibration, the guard crystals were treated in a slightly different manner. In order to avoid causing a bias in the gains due to a loss of energy out of the sides of these crystals into the tunnel, the centers of the $\gamma$ clusters were not allowed to be in one of these guard crystals. Calibrations were still achieved using clusters with centers in the regular crystals just outside the guard crystals, but the measured gains were expected to be less accurate.

### TABLE II. Summary of corrections to the beam normalization at each beam momentum, and the effective target length.

<table>
<thead>
<tr>
<th>(p$_{lab}$) (MeV/c)</th>
<th>Track reco. efficiency</th>
<th>f$_{TOF}$</th>
<th>f$_{target}$</th>
<th>f$_{decay}$</th>
<th>f$_{acc}$</th>
<th>f$_{WV}$</th>
<th>Total correction</th>
<th>Eff. Target Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>514</td>
<td>0.581±0.003</td>
<td>0.783±0.005</td>
<td>0.838±0.013</td>
<td>0.620±0.003</td>
<td>0.880±0.083</td>
<td>0.906±0.001</td>
<td>0.189±0.018</td>
<td>9.428±0.002</td>
</tr>
<tr>
<td>560</td>
<td>0.600±0.008</td>
<td>0.923±0.008</td>
<td>0.863±0.004</td>
<td>0.654±0.003</td>
<td>0.941±0.038</td>
<td>0.904±0.001</td>
<td>0.266±0.025</td>
<td>9.484±0.004</td>
</tr>
<tr>
<td>581</td>
<td>0.652±0.015</td>
<td>0.828±0.010</td>
<td>0.850±0.002</td>
<td>0.667±0.003</td>
<td>0.872±0.082</td>
<td>0.906±0.001</td>
<td>0.242±0.024</td>
<td>9.467±0.002</td>
</tr>
<tr>
<td>629</td>
<td>0.734±0.006</td>
<td>0.887±0.005</td>
<td>0.885±0.006</td>
<td>0.695±0.003</td>
<td>0.922±0.086</td>
<td>0.917±0.001</td>
<td>0.339±0.032</td>
<td>9.519±0.002</td>
</tr>
<tr>
<td>659</td>
<td>0.747±0.008</td>
<td>0.938±0.011</td>
<td>0.894±0.002</td>
<td>0.727±0.002</td>
<td>0.951±0.089</td>
<td>0.914±0.001</td>
<td>0.386±0.037</td>
<td>9.507±0.002</td>
</tr>
<tr>
<td>687</td>
<td>0.775±0.005</td>
<td>0.859±0.011</td>
<td>0.892±0.001</td>
<td>0.718±0.002</td>
<td>0.921±0.086</td>
<td>0.924±0.001</td>
<td>0.362±0.034</td>
<td>9.489±0.002</td>
</tr>
<tr>
<td>714</td>
<td>0.656±0.015</td>
<td>0.776±0.018</td>
<td>0.907±0.003</td>
<td>0.726±0.002</td>
<td>0.957±0.090</td>
<td>0.931±0.001</td>
<td>0.299±0.030</td>
<td>9.573±0.005</td>
</tr>
<tr>
<td>750</td>
<td>0.736±0.004</td>
<td>0.902±0.013</td>
<td>0.882±0.006</td>
<td>0.734±0.002</td>
<td>0.953±0.089</td>
<td>0.925±0.001</td>
<td>0.379±0.036</td>
<td>9.487±0.003</td>
</tr>
</tbody>
</table>

B. Beam tracks and scalers

The number of kaons striking the liquid-hydrogen target was monitored for most of the data using the GOOD-BEAM scaler described in Sec. IIE and modified by several corrections. These corrections account for such quantities as detector efficiencies, kaon decays in flight, accidental coincidences, and pions with the same time of flight as kaons. The corrections were determined for each run, if possible, at each beam momentum for both full- and empty-target conditions. They are summarized in the following sections and in Table II. The beam tracks were also used to estimate the effective target length.

Numerically, the number of useful kaons at the target can be described as

$$N = N_0 \times \epsilon_{DC} \times f_{TOF} \times f_{target} \times f_{decay} \times f_{acc} \times f_{WV},$$

where $N_0$ is the number of kaons in the beam from the GOOD-BEAM scaler, $\epsilon_{DC}$ is the efficiency of detecting and tracking beam particles in the drift chambers, $f_{TOF}$ is the correction for the fraction of beam pions that have approximately the same time of flight as kaons, $f_{target}$ is the fraction of kaons with trajectories projecting into the target, $f_{decay}$ is the fraction of kaons remaining in the beam at the target, correcting for decays in flight and scattering, $f_{acc}$ is the correction for the fraction of accidental coincidences, and $f_{WV}$ is the fraction of kaons with trajectories not associated with the beam halo.
1. Beam track reconstruction and efficiency

For the analysis of the $K^-p \rightarrow \Sigma^0\pi^0$ reaction, a reconstructed beam track was required. Thus, the track reconstruction efficiency was one correction to the GOOD-BEAM scaler. The beam tracks were important to test whether they projected to the target, to estimate the effective target length, and to eliminate events where an obvious scattering of the beam particle not related to the target occurred. A file of the tracks from the KBEAM trigger was made from one run at each momentum, and these were input to various simulation studies as described below.

Beam tracks were reconstructed with information from several drift chambers, $D_1$–$D_4$, and from the $S_1$ hodoscope. Each of the chambers $D_2$, $D_3$, and $D_4$ contained a total of four wire planes. Two adjacent wire planes, staggered by approximately one-half of a wire spacing, measured the $x$ coordinate, and the other two adjacent planes measured the $y$ coordinate. The number and locations of hits from each chamber were determined separately in both the $x$ and $y$ directions. With just two wire planes, chamber $D_1$ only measured the $x$ coordinate. A “hit” in a chamber in the $x$ direction required signals from a wire for each plane, and the difference between wire positions for each pair of parallel wire planes was required to be less than the wire spacing of 5.08 mm. In addition, the sum of the drift times from the hits on two wire planes was required to fall within a selected range, corresponding to the drift time from half of a wire spacing. These conditions were imposed to resolve the drift chamber left-right ambiguity and to reject events with multiple hits. Similar requirements were imposed for a hit in the $y$ direction. The $D_1$ chamber was located 6.92 m upstream of the target location, while the other drift chambers were located between 2.58 and 2.98 m upstream of the target along the beam ($z$) direction. There was a distance of about 4.3 mm between pairs of $x$ or $y$ wire planes, but a distance of 3.72 mm was between the adjacent $x$ and $y$ planes.

A good track was required to be reconstructed in both the $x$ and $y$ directions for the drift chambers $D_2$–$D_4$. Thus, at least two chambers must have a hit in $x$ and two chambers a hit in $y$, but the chamber pairs may have differed for the two directions. If hits occurred in all three chambers for a given direction, a track was defined by the triplet of hits with the minimum difference at $D_3$ from the line determined by the $D_2$ and $D_4$ hit positions. The track parameters were then determined by a least-squares fit to the three hits, these hits were removed from the list of hits, and the procedure was continued until no more three-hit tracks could be reconstructed. If any leftover hits remained from two chambers, all combinations of two-hit tracks were then found. Events were rejected if more than two tracks were found in $x$ or $y$. The average track parameters for the two tracks were adopted, because for most events of this type, the tracks were very close.

Chamber $D_1$ was treated differently because of the high singles rates due to its location near the mass-selection-slit collimator, and because of the nearby $S_1$ hodoscope, which had eight scintillator strips parallel to the $D_1$ wires. If there was a unique hit in $D_1$, that hit position was used for the final beam track, independent of the $S_1$ hodoscope information. For two $D_1$ hits and a single signal in an $S_1$ scintillator strip, the average chamber hit position was adopted if both hits matched the position of the strip, otherwise the event was rejected. For other multiple $D_1$ hits and a single $S_1$ scintillator strip signal, the event was kept only if a unique $D_1$ hit matched the $S_1$ strip position. Finally, if there was no good $D_1$ hit but a unique $S_1$ strip, a position in the scintillator strip was adopted. Note that the trigger was set to preclude multiple $S_1$ counter hits within a tight timing window, but the software tested for multiple hits in a wider window.

The last step in the beam track reconstruction combined the $x$ and $y$ track information from chambers $D_2$–$D_4$, the point near the mass slit, and the beam transport matrix through the $Q_0$, $B_2$, and $Q_7$ magnets to derive the fractional momentum difference, $\delta p/p$, from the nominal beam momentum. The nominal momentum was obtained from other studies with these data (Refs. [4,38–40]) and with previous measurements with the same beam-line magnets [41]. The average reconstruction efficiency at each momentum is recorded in Table II.

2. Kaon fraction

The fraction of the GOOD-BEAM scaler events that were kaons, $f_{\text{TOF}}$, was estimated from the $(S_1 - S_1)$ difference time-of-flight spectra. Examples are shown in Fig. 3 for one $S_1$ hodoscope counter at momenta of 514 and 750 MeV/c. Only five or six of the hodoscope counters detected significant numbers of events, depending on the beam momentum. The time spectra from each of the hodoscope counters were combined so that all peaks matched. A fit was made of this combined peak with a quadratic background, and the fraction of kaons was extracted from the total. The fraction $f_{\text{TOF}}$ varied from about 78% to 94%, depending on the beam momentum, as shown in Table II. The quoted uncertainty was estimated from the run-to-run variation of $f_{\text{TOF}}$.

3. Kaons missing the target

Events were retained only if they had a good beam track. This permitted the momentum of the incoming particle to be computed, as well as the projected interaction points, $x_I$ and $y_I$, at the target center ($z = 0$). A cut on the radial distance from the nominal beam line, $r_I = \sqrt{x_I^2 + y_I^2} < 5.0$ cm, was made to ensure the interaction occurred in the liquid hydrogen rather than in the target walls; see Fig. 5. The fraction of CB-NEUTRAL events passing this cut, $f_{\text{target}}$, is given in Table II. The uncertainty quoted was estimated from the run-to-run variation in this fraction.

4. Kaon decays and scattering

Some of the events scaled in GOOD-BEAM were kaons that decayed before reaching the target, since $S_T$ was a significant distance upstream of the CB center. A simple correction can be made for these decays. In addition, some GOOD-BEAM events were actually kaons that decayed before $S_T$, or were scattered in upstream material, causing them to miss the target. For example, a $K^- \rightarrow \mu^-\nu$ decay could have occurred upstream of $S_T$, with the $\mu^-$ detected in $S_T$, the average chamber hit position was adopted if both hits matched the position of the strip, otherwise the event was rejected. For other multiple $D_1$ hits and a single $S_1$ scintillator strip signal, the event was kept only if a unique $D_1$ hit matched the $S_1$ strip position. Finally, if there was no good $D_1$ hit but a unique $S_1$ strip, a position in the scintillator strip was adopted. Note that the trigger was set to preclude multiple $S_1$ counter hits within a tight timing window, but the software tested for multiple hits in a wider window.

The last step in the beam track reconstruction combined the $x$ and $y$ track information from chambers $D_2$–$D_4$, the point near the mass slit, and the beam transport matrix through the $Q_0$, $B_2$, and $Q_7$ magnets to derive the fractional momentum difference, $\delta p/p$, from the nominal beam momentum. The nominal momentum was obtained from other studies with these data (Refs. [4,38–40]) and with previous measurements with the same beam-line magnets [41]. The average reconstruction efficiency at each momentum is recorded in Table II.

2. Kaon fraction

The fraction of the GOOD-BEAM scaler events that were kaons, $f_{\text{TOF}}$, was estimated from the $(S_1 - S_1)$ difference time-of-flight spectra. Examples are shown in Fig. 3 for one $S_1$ hodoscope counter at momenta of 514 and 750 MeV/c. Only five or six of the hodoscope counters detected significant numbers of events, depending on the beam momentum. The time spectra from each of the hodoscope counters were combined so that all peaks matched. A fit was made of this combined peak with a quadratic background, and the fraction of kaons was extracted from the total. The fraction $f_{\text{TOF}}$ varied from about 78% to 94%, depending on the beam momentum, as shown in Table II. The quoted uncertainty was estimated from the run-to-run variation of $f_{\text{TOF}}$.

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to give a GOOD-BEAM count. As another example, some kaons that decayed downstream of ST resulted in charged particles that struck HV counters, and thus were not counted in GOOD-BEAM.

The correction for these effects was estimated in a simulation of the beam, especially the ST, HV, and WV counters, and the air, vacuum windows, and wire chambers. Beam tracks from the KBEAM trigger were input to the GEANT-based simulation program, starting 350 cm upstream of the target center; see Sec. III E. This was in the region of $S_2$ and the chambers $D_2$–$D_4$. The simulated events were tracked, and energies recorded in various counters. The number of GOOD-BEAM events was recorded for events with energy deposited in $S_2$ and ST, but not in HV. The ratio of the number of simulated kaons reaching the target center to the number of GOOD-BEAM counts gave the desired correction, $f_{\text{decay}}$; see Table II. Note that this correction is dominated by the fraction of kaons decaying in the 162 cm between ST and the target. The uncertainty is estimated from the spread in the measured beam momentum and the uncertainty in the detector positions.

A cross-check of the scattering contributions, beam decays, and accidentals can be made by comparing the projected beam-track position at ST to the time difference, $(S_{\text{TR}} - S_{\text{TL}})$, in the horizontal plane; see Fig. 6. The contribution from the good KBEAM events is located on the diagonal in Fig. 6; outside of this region are KBEAM events from other processes (see Sec. III B5). Approximately 14% of the beam events fall outside of this position and timing window, when the restriction is added that beam-particle trajectories are within 5 cm of the nominal beam line at the target. This fraction is consistent with $f_{\text{decay}}$, since some kaon decays will produce charged particles that miss the ST counter.

5. Accidentals

The high particle rates in the beam, due mostly to pions, caused inaccurate counting of the number of incident kaons in the GOOD-BEAM scaler through several processes. One example has one beam particle going through $S_1$ and a different one through the $S_2$–$S_T$ pair (or the $S_1$–$S_2$ pair and $S_T$). These particles could have passed through $S_1$ at a sufficiently different time so they would not be rejected in the KBEAM electronic logic, combined with conditions, such as scattering in beam-line material or beam-particle decays, or particle momenta far from the nominal value. A second example involves an inefficiency in the $S_1$ hodoscope from electronic dead time in the discriminators and small gaps between the individual counters. The inefficiencies would have caused too few multiple-particle events from being vetoed in KBEAM, also yielding too many GOOD-BEAM counts. A third example results in too few GOOD-BEAM counts, which is caused by multiple kaons in the same $S_1$ hodoscope counter, at nearly the same time and with similar trajectories, such that they would not have been distinguished by the wire chambers and electronics.

It was not possible to simulate all these processes accurately because of the lack of information on many of the beam and detector parameters. Thus, a number of studies were performed with the data in an attempt to correct for these accidental effects.

An estimate of the KBEAM accidental rate was made by counting the number of $S_2$ and 128-ns-delayed $S_T$ signal coincidences, and comparing this to regular $S_2$–$S_T$ coincidences. The fraction of good KBEAM events from these values ranged from 80% to 96%. Another estimate of the accidental veto
rate of good events was made from the coincidence of the 
good event signal with a 256-ns-delayed signal of the sum of 
signals from all veto counters. The fraction of good events 
from these values was typically 94–97%. An estimate of the 
total accidental fraction was found by averaging the values 
from these two methods, since they were not necessarily 
independent measurements. The uncertainty in this accidental 
fraction was taken as one-half of the difference between 
the two measured fractions and then scaled by the largest 
uncertainty-to-fraction ratio. The results are given in Table II; 
the accidental uncertainty is the dominant contribution to the 
overall normalization uncertainty.

Additional checks of the accidental rate were made. One 
check was discussed in Sec. III B4 above, using the correlation 
of the horizontal \( S_T \) position and the difference in timing 
between \( S_T \) and \( S_L \). Another check was made of the number 
of hits in the beam veto counters, \( BV \) and \( BVS \), located 
downstream of the CB, in events that were fully reconstructed 
as a \( \Sigma^0\pi^0 \). A hit, as measured by the TDCs for these beam-veto 
counters, occurred in 1.2% of the reconstructed \( \Sigma^0\pi^0 \) events. 
Accidental coincidences of a cosmic ray in the CB with a 
beam particle were removed with empty target subtraction; 
slight differences between the average beam intensity for full 
and empty target runs are expected to have negligible effect.

6. Beam veto counter (WV) correction

Signals from the WV beam veto counters, discussed in 
Sec. II D, were not included in the number of GOOD-BEAM-
scaled events. Consequently, too many beam particles were 
counted in this value. A correction to GOOD-BEAM was made 
by comparing scaled values of KBEAM in anticoincidence 
with the HV and WV counters and without the WV counters. 
This correction, \( f_{WV} \), ranged from 90% to 93%.

7. Effective target length

The effective target length \( L_{\text{eff}} \) is defined as the amount of 
liquid hydrogen that a beam particle traversed within the target. 
This quantity is used to find the number of target particles and 
is part of the calculation of the target constant. Since the target 
in the CB was a cylinder with two hemispherical ends, different 
target lengths may have been traversed by beam particles with 
different trajectories. The value of \( L_{\text{eff}} \) can be calculated from 
the beam particle’s projected horizontal and vertical positions at 
the target center and also from the divergences of the beam 
track. An average \( L_{\text{eff}} \) can then be computed from all the 
beam-particle trajectories and is based on the geometry of the 
target only.

Particles in the beam halo or those with large divergences 
may be eliminated by imposing a radial distance cut, corre-
responding to the target diameter. A cut of 5.0 cm on the 
radial distance from the target center was used. The only other 
requirement for beam particles used in the calculation was that 
good drift-chamber tracks from the KBEAM trigger were used to 
determine the position and divergence at the target center.

Several checks of this calculation were made. For example, 
varying the radial distance cut gave the appropriate expected 
changes in \( L_{\text{eff}} \). Also, for runs that had several beam veto 
counters removed from the trigger, \( L_{\text{eff}} \) also responded 
accordingly.

The average effective target length was calculated for 
each beam momentum and weighted by the number of good 
beam-particle triggers. The uncertainties were estimated from 
run-to-run variations at each momentum due to different beam 
conditions. The values of \( L_{\text{eff}} \) were reasonably consistent 
between data runs and between different beam momenta. The 
results are presented in Table II.

C. Efficiencies of the veto barrel counters

Since only neutral final-state particles are required, the 
efficiencies of the veto barrel scintillation counters are an 
important consideration in the event selection. As described 
in Sec. II D, a photomultiplier tube viewed each end of 
four veto barrel scintillators. Both photomultiplier-tube signal 
outputs from a single scintillator must exceed minimum 
energy thresholds to determine whether a charged particle was 
detected. This requirement would then designate the event as 
a charged- or neutral-trigger event. The same process was 
simulated in the Monte Carlo program. Particles traversing 
the veto barrel counter would deposit energy, and if that energy 
exceeded the thresholds at both ends of the scintillator, then 
the trigger would be considered charged. Thus, the response of 
the veto barrel counters could be modeled to match the 
experimental hardware response.

The veto barrel counters were calibrated [42] using a 
300-MeV/c \( \pi^+ \) beam and \( \pi^- \) \( p \) elastic-scattering events. The 
position where the \( \pi^+ \) hit the veto barrel and the energy 
deposited was determined from elastic scattering kinematics. 
The light attenuation in each scintillator was measured from 
a fit of the photomultiplier tube output as a function of the 
hit position of cosmic rays at several locations along the 
scintillator. The effective threshold of the veto barrel was then 
determined from the hit position, the light attenuation, and the 
energy.

Several studies were performed to determine the veto 
barrel efficiencies of detecting a charged particle [43,44]. In 
Ref. [44], the charge-exchange reaction, \( \pi^- \) \( p \rightarrow \pi^0 n \), with 
a 192-MeV/c pion beam was used to measure the energy-
threshold dependence of the detection probability of photons 
in the scintillator. This probability varies according to the hit 
position in the counter due to light attenuation.

These energy thresholds were incorporated in the Monte 
Carlo simulation for each photomultiplier tube. The threshold 
values ranged from 0.8 to 1.7 MeV. Simulated events were 
designated as either charged or neutral triggers, depending on 
whether the energy deposited in a scintillator exceeded the 
threshold for both photomultiplier tubes. Good agreement was 
found between data and Monte Carlo predictions using these 
veto barrel energy thresholds for the \( \pi^- \) \( p \rightarrow \pi^0 n \), \( \pi^- \) \( p \rightarrow 
\gamma n \), and elastic scattering reactions.

D. Crystal Ball analysis

The first step in the reconstruction of events in the CB was to 
group the NaI-counter hits into clusters of energy from a single
After pedestal subtraction and multiplication by the gains for each CB ADC, all NaI-counter hits were ordered by energy. If the maximum counter energy was less than 20 MeV, the event was rejected. Otherwise, the counter with the largest deposited energy greater than 20 MeV and all NaI counters which it touched (usually 12, but 11 if the counter touched a corner of the icosahedron, and fewer if it was a guard crystal) were assigned to a cluster. The sum of energy in the cluster was computed. The counters in the cluster were removed from the list of NaI hits, and the procedure continued until there were no remaining counters with energy greater than 20 MeV. Energy from a single NaI counter was included in only one cluster and was not shared with other nearby clusters.

The position of the cluster was calculated from an energy-weighted sum of the positions of all NaI crystals in the cluster. A logarithmic energy correction to the depth of the interaction within the NaI crystal was made to the cluster position. Initially, angles corresponding to the direction of the particle producing the cluster were determined from the positions of the preliminary interaction point of the beam particle at the target center \( (x_I, y_I, z_I = 0) \) and the corrected cluster position.

The time for each cluster in an event was defined by the TDC corresponding to the NaI counter at the cluster center. A good timing cut of width about 13 ns was applied; see Fig. 7. If the cluster failed the good timing cut, it was removed from further consideration. The cluster was retained if there was no TDC information or if any of the multiple TDC values satisfied the timing cut.

Only five- and six-cluster events passing the TDC cut were considered for the reactions under study. Several conditions were applied: (1) Events with more than one cluster at angle \( \theta > 150^\circ \) relative to the nominal beam direction were eliminated, as they often corresponded to showers from upstream \( K^- \) decays to \( \pi^0 \)'s. This condition removed 13% of data, but only 1.0% of otherwise reconstructable \( \Sigma^0\pi^0 \) events in a Monte Carlo simulation. (2) If all the cluster centers were near a plane through the nominal beam line, the event was rejected. In detail, if the root-mean-squared deviation of the azimuthal angle \( \phi \) of the clusters from the mean \( \phi \) was less than 17\(^\circ\), it was likely the event originated from a \( K^- \) decay to a \( \mu^- \) or \( \pi^- \); see the middle panel of Fig. 8. This cut removed approximately 81% of the raw data, but only an estimated 1.6% of the good events. (3) The cluster centers were required to be separated by \( \delta_{\text{min}} = [\text{no. counters in cluster 1} + (\text{no. counters in cluster 2}) - 1] \) in cm at the inside diameter of the CB, or typically 9 cm or 17\(^\circ\), in order to avoid cluster overlap problems, such as energy sharing. (4) Events with a photon cluster centered on a guard crystal were rejected. This guard crystal constraint reduced the number of good \( \Sigma^0\pi^0 \) candidates by almost a factor of 2. However, when measured without this constraint, the differential cross sections deviated significantly from the results presented here and varied systematically with \( \cos(\theta_{\text{cm}}) \). In contrast, a more restrictive fiducial cut exhibited no such distortion of the angular distribution. The differences in cross section were generally within a fraction of the statistical uncertainty for the restricted fiducial cuts except where the acceptance was very small (Fig. 9). The need to eliminate events with \( \gamma \) clusters in the guard crystals is believed to be related to imperfect pileup corrections and NaI gain calibrations. For the six-cluster case, events with a cluster designated as a neutron centered in a guard crystal were accepted, as the analysis makes use of only the neutron cluster angle and not the energy.

The remaining steps in the reconstruction of the \( \Sigma^0\pi^0 \) events were (1) a test to ensure there were at least two \( \pi^0 \) candidates in the event, with no common clusters, (2) evaluation of a \( \chi^2 \)-like function \( F \) expressed in terms of the interaction vertex \( z_I \) and the distance from that point to the decay vertex \( d_{1D} \), and (3) cuts on the minimum value of \( F \), the corresponding \( z_I \), and other kinematic quantities; see Table III.

For the first step of determining \( \pi^0 \) candidates, all combinations of energy clusters were combined in pairs. The invariant mass \( IM_{\gamma} \) was calculated for each pair to see if it matched the \( \pi^0 \) mass, assuming the clusters were from \( \gamma \)'s and an interaction point at \( (x_I, y_I, z_I = 0) \). A value in the range \( 115 \leq IM_{\gamma} \leq 160 \) MeV\(^2/c^2 \) was required to consider the cluster pair as a \( \pi^0 \) candidate. If there were fewer than two \( \pi^0 \) candidates from four distinct clusters, the event was rejected.

The function \( F \) was calculated from a variety of invariant and missing masses (MM), and weights. In all cases, the beam particle was assumed to be a kaon with momentum measured with wire chamber data, and the interaction-point coordinates transverse to the beam direction were assigned to be \( x_I \) and \( y_I \). For five-cluster events, each cluster was assigned to be \( \gamma_5 \) through \( \gamma_1 \) through \( \gamma_5 \), and \( F \) was computed from

\[
F = w_p \cdot (IM_{\gamma_5} - M_{\pi^0})^2 + w_D \cdot (IM_{\gamma_4} - M_{\pi^0})^2 + w_\Sigma \cdot (MM_{\gamma_4} - MM_{\pi^0})^2 + w_\Lambda \cdot (MM_{\gamma_5} - MM_{\pi^0})^2 + w_n \cdot (MM_{\gamma_5} - M_n)^2.
\]
TABLE III. Events remaining after cuts in the analysis of both data and Monte Carlo simulations at 659 MeV/c. The number of Monte Carlo events in the first four columns was chosen to match total cross sections from Ref. [7]. The “Filter” cuts include >4 clusters, <2 clusters beyond 150°, and clusters outside of a plane containing the beam direction. The “Guard and other cuts” require all photon cluster candidates to pass the guard crystal cut, to have at least one π^0 with missing mass approximately equal to the Σ^0 mass, and to have a second π^0 formed from different clusters; these entries are the sum of the five- and six-cluster events in the following sections of this table. The dominant loss of events for this category is from the guard crystal cut.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Σ^0π^0</th>
<th>Λπ^0</th>
<th>Λπ^0π^0</th>
<th>Σ^0K^0n</th>
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</thead>
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<td>9145</td>
<td>18917</td>
<td>11176</td>
<td>87939</td>
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<td>8855</td>
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<td>10957</td>
<td>75471</td>
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<td>6868</td>
<td>16561</td>
<td>9031</td>
<td>53761</td>
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<td>≥ 2 unique π^0</td>
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<td>5803</td>
<td>15101</td>
<td>8013</td>
<td>39860</td>
</tr>
<tr>
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<td>2388</td>
<td>9103</td>
<td>3301</td>
<td>13293</td>
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<td>3042</td>
<td>8443</td>
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<td>144</td>
<td>143</td>
<td>121</td>
<td>2490</td>
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<tr>
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<td>73</td>
<td>53</td>
<td>98</td>
<td>2230</td>
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<td>44</td>
<td>82</td>
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<tr>
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<tr>
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<td>400</td>
<td>6</td>
<td>17</td>
<td>5</td>
<td>399</td>
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</table>

For six-cluster events, the sixth cluster was assumed to be from the neutron, and the last term in Eq. (3) was replaced by

$$w_{\text{mom}} \cdot (p_{\Lambda_1} - p_{\Lambda_2})^2 + w_{\text{ang}} \cdot (\cos^{-1}(\hat{p}_{\Lambda_1} \cdot \hat{p}_{\Lambda_2} / p_{\Lambda_1} p_{\Lambda_2}))^2.$$

In the expressions above, the π^0, Σ^0, Λ, and neutron masses are M_{π^0}, M_{Σ^0}, M_Λ, and M_n, respectively. The weight w_P applies to the invariant mass of the pion produced at the interaction vertex, where the Σ^0 is also produced. The pion decays to γ_1 and γ_2, and is referred to as the “production pion.” The weight w_D corresponds to the invariant mass of the pion formed from γ_1 and γ_2, which comes from the decay of the Λ^0. Similarly, the weights for the Σ, Λ, and neutron terms are w_{Σ}, w_{Λ}, and w_n, respectively. The momentum vector \(\hat{p}_{\Lambda_1}\) for the Λ^0 was computed from the vectors associated with the interaction vertex, namely, \(\hat{p}_{\Lambda_1}\), \(\hat{p}_{\gamma_1}\), and \(\hat{p}_{\Lambda_1}\) (from the radiative photon), and the initial state of the beam kaon and the target proton. The vector \(\hat{p}_{\Lambda_2}\) was computed from the vectors associated with the decay vertex (\(γ_1, γ_2\), and the angles of the sixth cluster) assuming these originated from a Λ^0 decay. The weights for the magnitude of the momentum difference and for the angle difference are \(w_{\text{mom}}\) and \(w_{\text{ang}}\), respectively.

All the weights were determined from Monte Carlo simulations. For example, \(1/w_w\) was the square of the standard deviation for the MM_{γ_1γ_2γ_3γ_4γ_5} distribution after a cut on \(F\) with that term removed, and similarly for the other weights. This process of determining the weights was iterated several times until the weights were determined to be stable. The weights were calculated at each momentum and were slowly varying with energy.

All combinations of cluster assignments were tried in Eq. (3). A \(F\) value was computed for each one, and the correct assignment was assumed to be that giving the minimum value of \(F\). For about 30% of the events, the assignment with the second best \(F\) value passed all the cuts described below, and 4% of the third best passed the cuts. In 80% of the second best \(F\) assignments, γ_1 and γ_2 had the same cluster identity, yielding the same scattering angle, \(θ_{c.m.}\); hence the cross section was not affected. For the second best \(F\) assignments in which different photons were associated with the production pion, cos(θ_{c.m.}) typically changed by less than the angular resolution; hence the cross section would be little affected.

The final event selection was made by a cut on the minimum value of \(F\) (see Fig. 10) and on the fitted interaction vertex z_I (Fig. 11). These two cuts were chosen to maximize the signal-to-background ratio. In addition, some very loose cuts were imposed on the quantities, IM_{γ_1γ_2}, MM_{γ_1γ_2}, MM_{γ_1γ_2γ_3γ_4γ_5}, and MM_{γ_1γ_2γ_3γ_4γ_5γ_6}. Figure 12 shows MM_{γ_1γ_2} before the cut on \(F\), after the \(F\) cut, but before those on the four invariant and missing masses, and after all cuts. Finally, Fig. 13 presents four invariant and missing masses after all cuts along with simulation results that are normalized to give the same total number of events.

E. Simulations and acceptance

The detector acceptance was obtained from a Monte Carlo simulation based on the GEANT 3.21 code [45]. The input geometry consisted of all experimental detectors downstream of the last beam-line magnet Q_2. In addition, the beam collimators and parts of the beam shielding, NaI detector dryroom walls, beam pipe, and liquid-hydrogen target apparatus were included. Some of these items were unimportant for the acceptance calculations, but were used to estimate corrections to the number of beam counts, etc.; see Sec. III B. A cut on ΣCB was made for Monte Carlo events to simulate the trigger condition, but all simulated Σ^0π^0 events passed this cut. Also, the energy deposited in each NaI crystal was corrected in the Monte Carlo simulations to account for the small (average) energy lost outside the cluster boundary, out the back of the crystals, and between the crystals. The value of the energy loss correction (6.5%) was determined by reproducing measured IMs and MMs and the reconstructed interaction vertex z_I, and is independently consistent with the Σ^0γ analysis result [46].

An iterative procedure was used to calculate the acceptance. The \(K^- p \rightarrow Σ^0π^0\) events were initially generated isotropically, the acceptance computed, and the differential cross section was then determined from the data. Events were generated from this differential cross section, and the acceptance computed again. At forward and backward angles, the acceptance changed significantly because of finite angular resolution and steeply changing cross sections. A third iteration of this procedure at two sample energies resulted in very small changes to the acceptance compared to statistical uncertainties on the data. Therefore, the acceptance from two iterations was adopted.
FIG. 8. (Color online) Displays of single events showing the energy deposited in each crystal for (upper) a good $K^− p → Σ^0 π^0$ candidate, (middle) a $K^−$ decay to a $μ^−$ or $π^−$ upstream of the CB, and (lower) an example of energy from outside the good timing window. Individual counters outlined in red are part of an identified $γ$ cluster that passed the timing cut.
FIG. 9. Comparison of the angular distribution for the $K^- p \rightarrow \Sigma^0 \pi^0$ reaction with two different fiducial cuts at 659 MeV/c laboratory momentum. The standard cut removed $\gamma$ clusters centered in a guard crystal, while the restricted cut required the clusters to have $39^\circ \leq \theta_\gamma \leq 141^\circ$.

The target interaction points used in the simulation were chosen from data events as follows. Beam tracks from the sampled KBEAM triggers were projected to the center of the CB ($z = 0$), and the transverse position calculated. The location along the beam ($z$ direction) was chosen uniformly within the target volume, ignoring the small beam attenuation effects. Typically, 50 000 KBEAM trigger events were used at each beam momentum from the data runs.

FIG. 10. Distribution of the minimum value of $F$ from Eq. (3) for five-cluster events at 659 MeV/c beam momentum, with the arrow indicating the cut to reduce backgrounds.

FIG. 11. Distribution of the derived interaction vertex position $z_I$ from data at 659 MeV/c beam momentum after subtraction of empty-target events. The cuts applied to the data are indicated. Monte Carlo simulations of the $z_I$ distribution including backgrounds and pileup, normalized to the same total number of events, are included for comparison as the open circles.

The $\Lambda$ tracks from the $\Sigma^0$ decay were propagated in GEANT until their simulated decay. Neutron interactions in the NaI counters were simulated by the FLUKA subroutine in GEANT (see Ref. [47]). The energy deposited in each NaI and scintillation counter was recorded. Corrections for

FIG. 12. Distribution of the missing mass $M_{\gamma_1\gamma_2}$ is shown before the $F$ cut, after the $F$ cut but before mass cuts, and after all cuts. The data are for 659 MeV/c beam momentum.
light attenuation effects in the VB counters were included, as described in Sec. III C. For the acceptance calculations, the VB counters were the only scintillation counters whose signals were considered, since the HV counters were included in both the GOOD-BEAM and CB-NEUTRAL signals, and corrections were applied to GOOD-BEAM for the WV counters; see Sec. III B6.

The significant “pileup” of energy in the NaI counters was also incorporated in the computer simulation. This pileup was caused by interactions of other beam particles close in time to the kaon interaction that caused the trigger for a particular event. This effect was reduced somewhat because information from the multihit TDCs was used to reject clusters that were outside the standard timing window. This solution was imperfect because two particles could arrive close in time so that both produced clusters within the good timing window. Furthermore, because of the long decay time of pulses from the NaI crystals, a measurable signal was often produced by residual scintillation light from particles striking a crystal before the earliest time that could be recorded by the TDCs. This pileup is believed to be the source of much of the 5–10 MeV signals seen in a typical event, predominantly near the beam entrance and exit (see Fig. 8). If such energy was located within a crystal that was also struck by a photon in the event of interest, then that piled-up energy would have been added to the true photon energy. This additional energy could interfere with event reconstruction.

Energies measured in CB-NEUTRAL data events were used to add backgrounds to the simulations. This procedure simulated the piled-up energy in the NaI crystals for Monte Carlo events. Adding energies from the CB-NEUTRAL events in the manner described below provided a better simulation of real data events than adding energies from the PULSER trigger events with no beam, since these latter events exhibited a much lower background than the events with beam. The resulting simulated events also compared favorably to real data when inspected on an event-by-event basis with the single event display.

The goal of the pileup simulation procedure was to extract from the real CB-NEUTRAL events all observed energy that was not correlated with the kaon that triggered the event. Most of this background can be estimated as the energy from minor triangles that do not have a TDC hit within the good timing window. Therefore a randomly chosen CB-NEUTRAL event at the same beam energy as the simulation was inspected, and all energy deposited in minor triangles that had no TDC hit within the good timing window was added crystal-by-crystal to the simulated energy produced by \textsc{geant} for the $\Sigma^0\pi^0$ reaction. This energy is normally excluded from the analysis by the TDC cut as described in Sec. III D. It was important that the times recorded by the TDC from the data be added to the Monte Carlo events, so that the added energy would be handled similarly for data and Monte Carlo events. Therefore the added energy would usually affect the event reconstruction only if it overlapped energy produced by a photon in the Monte Carlo simulations.

In the procedure described so far, background clusters within the good timing window have not been simulated. To estimate the effect of this background, a second randomly chosen CB-NEUTRAL event at the same beam energy was inspected to find energy to be added to the simulation. However, the only energy added from this second event came from minor triangles having a TDC hit within a window of the same width as the good timing window, but offset from it. Before the Monte Carlo event was reconstructed, these triangles were assigned TDC hits within the good timing window. Four different offset time windows were tested, two before the good timing window and two after, and similar pileup results were obtained.

As part of the procedure to add background energy to the Monte Carlo, the energy produced in \textsc{geant} was inspected. For each crystal with simulated energy greater than 10 MeV, an additional simulated TDC hit within the good timing window was added to that crystal’s minor triangle. This allowed the timing cut to have the proper effect in the analysis of Monte Carlo data.

To summarize, background in the Crystal Ball was simulated in the Monte Carlo by adding part of the energy from two distinct, real CB-NEUTRAL events to that produced in \textsc{geant}. TDC hit data, some from the real events and some simulated, were also passed on to the reconstruction software. The resulting simulated events were analyzed with exactly the same software and the same cuts, including timing cuts and handling of multiple TDC hits, that were used for data.

The final acceptance was the ratio of the simulated events passing the cuts to those generated at each center-of-mass angle bin. The addition of the pileup energies typically reduced the acceptance by 15–20%. The acceptance calculated at 659 MeV/c is shown in Fig. 14.
the data. They varied from 2% to 20% as a function of angle at 750 MeV/c and from 1% to 16% at 659 MeV/c.

As a cross-check, the distribution of the $\Lambda$ lifetime was computed from the five-cluster data at 659 MeV/c. The results agreed within statistical uncertainties with simulations including kaon-induced backgrounds and pileup.

**G. Determination of the $\Sigma^0$ polarization**

The measurement of the $\Sigma^0$ polarization in exclusive channels is difficult when only charged particles are detected. For example, in hydrogen bubble chambers the $K^- p \rightarrow \Sigma^0\pi^0$ reaction was detected only via the $\Lambda \rightarrow p\pi^-$ decay, and the $\Sigma^0$ polarization was inferred from the $\Lambda$ polarization; see Ref. [48]. In fact, the very large statistical uncertainties on the $\Sigma^0$ polarization results from past experiments have provided very limited constraints on partial-wave analyses. For example, London et al. note [9], “We estimate that at least 2000 events at each energy would be required for a good polarization measurement.” In this experiment, the $\Sigma^0$ direction and momentum were reconstructed, allowing improved polarization determination with on average more than 2000 events at each momentum (see also Ref. [49]).

The $\Sigma^0$ polarization direction is perpendicular to the scattering plane or along the normal $\vec{q} = (\vec{p}_K \times \vec{p}_\Sigma)/|\vec{p}_K \times \vec{p}_\Sigma|$. The magnitude of the $\Lambda$ polarization is given by (Ref. [49], see p. 1144)

$$P_\Lambda = -P_\Sigma \cos \theta_\Lambda,$$

where $\theta_\Lambda$ is the angle between the $\Sigma^0$ polarization direction and the $\Lambda$ momentum in the $\Sigma^0$ rest frame. Also in the $\Sigma^0$ rest frame, the $\Lambda$ polarization is along its direction of motion. The angular distribution of the $\Lambda$ decay is

$$P = \frac{1}{2}(1 + \alpha_\Lambda P_\Sigma \cos \theta_\Lambda \cos \theta_n),$$

where $\theta_n$ is the angle between the $\Lambda$ polarization and the neutron momentum in the $\Lambda$ rest frame, and $\alpha_\Lambda = 0.642 \pm 0.013$ [5]. Thus, the decay-angle distribution for the $\Sigma^0 \rightarrow \gamma\Lambda \rightarrow \gamma n\pi^0$ decay is (see Ref. [49])

$$P = \frac{1}{2}(1 - \alpha_\Lambda P_\Sigma \cos \theta_\Lambda \cos \theta_n).$$

Hence

$$\langle \cos \theta_\Lambda \cos \theta_n \rangle = -\alpha_\Lambda P_\Sigma / 9 = \frac{1}{N} \sum_1^N \cos \theta_\Lambda \cos \theta_n,$$

(5)

The statistical uncertainty can be evaluated using the technique described by Solmitz [50] and gives

$$\delta P_\Sigma = \frac{1}{\alpha_\Lambda} \sqrt{\frac{9 - \alpha_\Lambda^2 P_\Sigma^2}{N}}.$$  

(6)

This result is approximately $\sqrt{3}$ smaller than the corresponding equation used for the hydrogen bubble-chamber measurements [7].

The above equations assume a perfect detector with acceptance over $4\pi$ sr; however, the Crystal Ball does not have full acceptance. Figure 15 shows the measured distribution

**F. Background estimation**

In addition to pileup effects, there were also backgrounds caused by beam interactions in the target flask and insulation and by other $K^- p$ reactions in the liquid hydrogen. Empty-target runs with cold gas in the flask were taken at each momentum. Events from these runs were analyzed with the same data cuts as the full-target runs and subtracted from the number of good events. The empty-target background was typically 1.6–5.1% of the full-target events. The very small effect of residual gas in the target flask during empty-target runs was ignored.

Although corrections to the GOOD-BEAM scaler were made for the number of pions in the beam, such beam particles could have led to events that mimic the $K^- p \rightarrow \Sigma^0\pi^0$ reaction. This possibility was tested by analyzing data with pion beams at the same beam momentum taken with the Crystal Ball. For equal numbers of CB-NEUTRAL triggers recorded with $\pi^-$ and $K^-$ beams, only 0.2% as many events from $\pi^-$ data compared to $K^-$ data were successfully reconstructed to the reaction $K^- p \rightarrow \Sigma^0\pi^0$. As the pion content of the beam within the TDC window (Fig. 3) is small, a negligible number of the reconstructed $\Sigma^0\pi^0$ events are produced by $\pi^-$ interactions.

Finally, kaon-induced backgrounds were studied. Events were generated from measured differential cross sections for $K^- p \rightarrow K^0\pi^- n$ and $\Lambda\pi^0$, and according to the phase space for $K^- p \rightarrow \Lambda\pi^0\pi^0$. Total cross sections were taken from Armenteros et al. [7]. These events were simulated by GEANT, including pileup, and then processed with the data analysis program. The final cuts applied to all the data were partially chosen based on these simulated events, in order to maximize the signal-to-noise ratio. After suitable normalization, the background events were subtracted at each angle bin from
of \(\cos \theta_A\) for two beam momenta. Because the \(\Sigma^0\) decays electromagnetically, the distribution of the \(\Lambda\) should be uniform and independent of the \(\Sigma^0\) spin direction. However, the momentum of the radiative photon (from the decay of the \(\Sigma^0\)) is tightly correlated with the measured \(\cos \theta_A\), and because the event is not reconstructed when the radiative photon leaves through the exit or entrance tunnels, the acceptance is diminished for \(\cos \theta_A \approx 0\).

The derivation can be repeated for the case of a real detector with imperfect event reconstruction. The probability distribution for the \(K^-p \rightarrow \Sigma^0\pi^0\) reaction to produce decay angles \(\theta_A\) and \(\theta'_B\) and be fully reconstructed with measured decay angles \(\theta_A'\) and \(\theta'_B'\) can be written as

\[
P = \frac{1}{2}(1 - \alpha_A P_2 x_\Lambda x_n) A(x_\Lambda, x'_\Lambda, x_n, x'_n).
\]

To simplify the expression, the substitutions \(x'_n = \cos \theta'_B\) and so forth have been used. The arbitrary acceptance and resolution function \(A\) can be expanded in terms of Legendre polynomials to

\[
A(x_\Lambda, x'_\Lambda, x_n, x'_n) = \sum_{jklm} A_{jklm} P_j(x_\Lambda) P_k(x'_\Lambda) P_l(x_n) P_m(x'_n).
\]

Following the earlier derivation, one obtains

\[
P_\Sigma = \left( \frac{1}{\alpha_A} \right) \frac{9 A_{0000} (x'_\Lambda x'_n) - A_{1010}}{A_{1010} (x'_\Lambda x'_n) - A_{1111}/9} P_\Sigma',
\]

The statistical uncertainty can be computed as

\[
\delta P_\Sigma = \frac{(1/\alpha_A)}{\sqrt{\delta(x'_\Lambda x'_n)}},
\]

with

\[
\delta(x'_\Lambda x'_n) = \frac{1}{\sqrt{N}} \langle \delta(x'_\Lambda x'_n) \rangle,
\]

\[
\langle \delta(x'_\Lambda x'_n) \rangle^2 = \langle (x'_\Lambda x'_n)^2 \rangle - \langle x'_\Lambda x'_n \rangle^2,
\]

\[
\langle (x'_\Lambda x'_n)^2 \rangle = \frac{1}{9} \left( A_{0000} + \frac{2}{5} (A_{0002} + A_{0200}) + \frac{4}{25} A_{2020} - \frac{\alpha_A P_\Sigma}{9} \left( A_{1010} + \frac{2}{5} (A_{1012} + A_{1210}) + \frac{4}{25} A_{1212} \right) \right).
\]

The coefficients in the expansion of \(A\) were estimated from simulations. The standard Crystal Ball Monte Carlo tracking software based on GEANT 3.21 was used as described in Sec. III C, but the event generator was modified to allow simulations of polarized \(\Sigma^0\) events. The standard Monte Carlo as presently written does not simulate the effects of polarization on the decay of the \(\Lambda\), nor does it allow the kinematics of the \(\Lambda\) decay to be determined on an event-by-event basis. To permit simulation of the angular distribution of the \(\Lambda\) decay, this decay was simulated in the event generator, and the GEANT simulation began tracking the resulting neutron and photons from the vertex of the simulated \(K^-p\) interaction.

Uncertainties in the coefficients of \(A\) were estimated from their variation among subsets of simulated events. At each momentum, at least 14 sets of 120 000 events were simulated. The coefficients were calculated separately for each subset, and the uncertainty in each coefficient was estimated based on the standard deviation of that coefficient computed for each of the 14 sets. Although all coefficients and the full formulas above were used to compute the polarization and uncertainties, only the following three coefficients were significantly different from zero: \(A_{0000}, A_{1111},\) and \(A_{0200}\). Thus the expression for the polarization could be simplified to read

\[
P_\Sigma = \left( \frac{81}{\alpha_A} \right) \frac{A_{0000}}{A_{1111}} (x'_\Lambda x'_n),
\]

and Eq. (9) could be simplified to

\[
\langle (x'_\Lambda x'_n)^2 \rangle = \frac{1}{9} \left( 1 + \left( \frac{2}{5} \right) \frac{A_{2020}}{A_{0000}} \right).
\]

Furthermore, because the fractional uncertainty in \(A_{0000}\) and \(A_{1111}\) is approximately 2% or less at all angles and all momenta, the contribution from the Monte Carlo statistics to the overall uncertainty in the polarization measurement has been neglected.

The unphysical lifetime of the \(\Lambda\) used in this simulation is one contribution to the overall systematic uncertainty in the polarization measurement of \(\pm 0.05\). This is negligible in comparison to the statistical uncertainty. The systematic uncertainty was estimated from a number of studies, including some that used the event generator written for \(\Sigma^0\pi^0\) production with polarization. For these studies, the event generator was
extended to compute the decay angles, $\cos \theta_L$ and $\cos \theta_n$, from the produced photons after application of acceptance cuts and resolution functions. A reconstructed polarization was then computed using Eqs. (5) and (6). There was no attempt to simulate the photon interaction with the apparatus or the effect of the fitting methods described in the previous section. This simplified Monte Carlo simulation made it possible to test the difference between the acceptance effects for the case where the physical $\Lambda$ lifetime was used and the case where it was set to zero. This simulation was tested with a variety of kinematic and acceptance cuts and resolution functions. One series of tests eliminated those events from the reconstruction in which the $\Lambda$ reached the target wall. This was an extreme test of how interactions of the propagating $\Lambda$ would affect the results. The polarizations computed under different treatments of the $\Lambda$ propagation never differed by significantly more than the systematic uncertainty of $\pm 0.05$.

The weak dependence of the polarization on the $\Lambda$ propagation distance is related to the weak dependence of the acceptance on $\cos \theta_L$. The directions of photons from the decay of a slow-moving $\pi^0$ are not very sensitive to the $\pi^0$ direction in the sense that a gap in the nearly spherical acceptance of the Crystal Ball does not result in a corresponding lack of acceptance for a narrow range of $\pi^0$ momenta. However, as explained earlier, the loss of the $\Sigma^0$ decay photon through the entrance or exit tunnels of the Crystal Ball does result in a decreased acceptance for $\cos (\theta_L) \sim 0$.

The polarizations were corrected for backgrounds due to the target windows and to $K^0\eta$, $\Lambda\pi^0$, and $\Lambda\pi^0\pi^0$ production using the relationship

$$N_{\text{tot}}P_{\Sigma} = N_{\text{signal}}P_{\Sigma} + \sum_i N_i P_i,$$

where $N_i$ is the estimated number of reconstructed background events computed for the cross-section measurement (Sec. III F). The quantity $P_i$ is the polarization of the “$\Sigma^0$” extracted from events either in empty-target runs or in the simulated background channels that were successfully reconstructed using the hypothesis of a $\Sigma^0\pi^0$ event. The index $i$ indicates the empty-target case or the three background reactions, and the summation is over all four cases.

Although the statistical uncertainties in the background contributions were typically large, $\delta P_i > 1$, the backgrounds contribute a small number of events and thus do not dominate the overall statistical uncertainty. To study the possible effects of these backgrounds, the polarization of the $\Lambda$ in the reactions $\Lambda\pi^0$ and $\Lambda\pi^0\pi^0$ was simulated at two beam momenta for $P_\Sigma = 0, \pm 1$. The reconstructed $\Sigma^0$ polarization was not affected by these background polarizations, and thus no corrections to $P_\Sigma$ were applied.

### IV. EXPERIMENTAL RESULTS

#### A. Differential cross section for $K^- p \to \Sigma^0\pi^0$

The differential cross sections were computed using the number of good events after background subtractions, the corrected number of incident kaons, and the effective target length. The cross sections are given in Table IV with statistical uncertainties at the eight beam momenta. The quoted uncertainties include estimated contributions from subtracting backgrounds due to the empty target and pion- and kaon-induced reactions. Overall systematic uncertainties of $\pm 10\%$ are estimated from the uncertainty on the total correction factor to the GOOD-BEAM scaler, and the uncertainties in the effective target length and target dimensions ($\pm 2\%$); see Table II. The size of the $\cos(\theta_m)$ bins was chosen to match approximately the angular resolution [$\Delta \cos(\theta_m) \simeq \pm (0.05$–$0.06)$].

The results are plotted in Fig. 16 with earlier data from Refs. [7,8,10]. Reasonably good agreement is found at all angles and momenta. However, the Dombeck et al. [8] cross-section results are systematically larger and the Baxter et al. [10] values are somewhat smaller than data from this experiment. The Armenteros et al. [7] angular distributions are more flat at a number of momenta than those presented in this paper.

A study was performed at 659 MeV/c to determine the sensitivity of the cross-section data to specific partial waves. Partial-wave amplitudes from Gopal et al. [18] were used to predict the differential cross section, removing one partial-wave contribution at a time (see also Ref. [51]). The largest differences occurred when removing either the $S_{01}$ or $D_{03}$ partial waves, while little change was observed for the $D_{05}$, $P$, $F$, or $G$ waves. In particular, the “bowl” shape in the differential cross sections is found to be characteristic of a strong $D$-wave contribution, and therefore an indication that the $\Lambda(1690)$ plays a dominant role in this kinematic region. As
The measurements from Refs. [6,7] were made by detecting the masses, widths, and couplings for the $\Lambda(1670)^{-}\frac{1}{2}$ ($S_{01}$) and especially the $\Lambda(1690)^{\frac{1}{2}}$ ($D_{03}$) resonances, but probably relatively little about the $\Lambda(1600)^{+}\frac{1}{2}$ state. A new partial-wave analysis using these data is in progress [52].

Weighted Legendre polynomial fits to the data,

$$\frac{d\sigma}{d\Omega} = \sum_{l=0}^{3} A_l P_l(\cos\theta_{c.m.}),$$

were performed using $\cos(\theta_{c.m.})$ bins of width 0.2, and the results are given in Table V. (Fits were also made with $l = 0$ to 4, but the $A_4$ coefficients were consistent with zero at all momenta.)

The total cross section at each momentum was obtained by integrating the fitted differential cross section, yielding $4\pi A_0$, which is plotted in Fig. 17 for all measured momenta. An overall 10% systematic uncertainty is associated with the normalization of our cross sections. The systematic uncertainty in the total cross section due to integrating the fitted shape into the region of the beam exit hole was studied and found to be approximately 2%.

The fitted Legendre shapes for the four lowest momenta are consistent with those obtained from a partial-wave analysis [52] of all available $K^- p \to \Sigma\pi$ data (including charged-particle final states). The partial-wave analysis indicates a slight downturn in the forward direction at 659 and 687 MeV/c, which becomes flat by 714 MeV/c, and changes to a rising forward cross section by 750 MeV/c. Adding a fifth term to the Legendre series brings the extrapolated shape in the forward direction into closer agreement with that found in the partial-wave analysis, but it changes the total cross section by less than 2%. Figure 16 shows the fitted Legendre shape for the first four Legendre terms at all eight momenta for consistency.

For comparison, the results in Refs. [6–10,13] are also shown in Fig. 17. The various experiments are generally consistent among each other with the exception of the data of Baxter et al. [10], which appear too low. Note that the measurements from Refs. [6,7] were made by detecting only charged particles from the $\Lambda \to \pi^- p$ decay, using $\text{MM}(\pi^- p) > M_{\pi^-}\Lambda$ to select the desired events. However, backgrounds from $K^- p \to \Lambda\pi^0\pi^0\pi^0$ led to problems in extracting the $K^- p \to \Sigma^0\pi^0$ events in those experiments.

The opening of the $K^- p \to \Lambda\eta$ channel at $\sqrt{s} = 1663$ MeV, corresponding to a $K^-$ beam momentum of 722 MeV/c, can produce a “cusp” in the cross section at backward angles, i.e., at large negative $\cos(\theta_{c.m.})$. The differential cross sections at $\cos(\theta_{c.m.}) = -0.9$ for our data and those of Armenteros et al. [7] and Baxter et al. [10] are shown in Fig. 18. The increase in cross section near 722 MeV/c is suggestive of such a cusp effect. Note that the momentum spread of the 714 MeV/c data in this paper is $\pm 30$ MeV/c from Table I.

### B. $\Sigma^0$ polarization

The polarization results are given in Table VI and Fig. 19. Coefficients for Legendre polynomial fits to the data

<table>
<thead>
<tr>
<th>Momentum</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$\chi^2$/n.d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>514</td>
<td>0.1822±0.0127</td>
<td>−0.120±0.035</td>
<td>0.124±0.034</td>
<td>−0.028±0.040</td>
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</tbody>
</table>

### Table V. Coefficients of Legendre polynomial fits to the differential cross sections as a function of beam momentum. The coefficients are given in mb/sr, and momenta in MeV/c.

<table>
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### Table VI. $\Sigma^0$ polarization and statistical uncertainties as a function of $\cos(\theta_{c.m.})$ of the production pion relative to the beam direction. The beam momenta are given in MeV/c.

- For comparison, the results in Refs. [6–10,13] are also shown in Fig. 17. The various experiments are generally consistent among each other with the exception of the data of Baxter et al. [10], which appear too low. Note that the measurements from Refs. [6,7] were made by detecting only charged particles from the $\Lambda \to \pi^- p$ decay, using $\text{MM}(\pi^- p) > M_{\pi^-}\Lambda$ to select the desired events. However, backgrounds from $K^- p \to \Lambda\pi^0\pi^0\pi^0$ led to problems in extracting the $K^- p \to \Sigma^0\pi^0$ events in those experiments.

- The opening of the $K^- p \to \Lambda\eta$ channel at $\sqrt{s} = 1663$ MeV, corresponding to a $K^-$ beam momentum of 722 MeV/c, can produce a “cusp” in the cross section at backward angles, i.e., at large negative $\cos(\theta_{c.m.})$. The differential cross sections at $\cos(\theta_{c.m.}) = -0.9$ for our data and those of Armenteros et al. [7] and Baxter et al. [10] are shown in Fig. 18. The increase in cross section near 722 MeV/c is suggestive of such a cusp effect. Note that the momentum spread of the 714 MeV/c data in this paper is $\pm 30$ MeV/c from Table I.

- The polarization results are given in Table VI and Fig. 19. Coefficients for Legendre polynomial fits to the data...
are presented in Table VII. The Legendre coefficients were obtained by fitting the polarization data to the expression

\[ P_\Sigma(\cos \theta) = \frac{\sum_{n=1}^{3} \frac{b_n}{A_n} P_n(\cos \theta)}{1 + \sum_{n=1}^{3} \frac{a_n}{B_n} P_n(\cos \theta)}, \]

with the ratios \( B_n/A_n \) treated as free parameters and the ratios \( A_n/A_0 \) fixed at the values given in Table VI. The fits were not constrained to give polarizations between \(-1 \) and \(+1\), and thus the central values of some of the fits give unphysical results, especially at 560 and 687 MeV/c. The fits were done keeping three terms. But as can be seen from Table VII, the \( B_2 \) and \( B_3 \) coefficients are consistent with zero. However, we keep them for the comparison to Gopal’s predictions below.

The behavior of the Legendre coefficients in Fig. 20 in terms of partial waves can be understood as follows. In the notation of Gopal et al. [18],

\[ \frac{d\sigma}{d\Omega} = \frac{k'}{k} [\|f\|^2 + |g|^2], \]

\[ P \frac{d\sigma}{d\Omega} = \frac{2k'}{k} \text{Im}[fg^*], \]

with \( k \) and \( k' \) the incoming and outgoing c.m. momenta, and with the nonflip and spin-flip amplitudes

\[ f = \frac{1}{\sqrt{kk'}} \left( T_{00} P_0(\cos \theta) + [2T_{1+} + T_{1-}] P_1(\cos \theta) + [3T_{2+} + 2T_{2-}] P_2(\cos \theta) + \cdots \right), \]

\[ g = \frac{1}{\sqrt{kk'}} \left( [T_{1+} - T_{1-}] P_1(\cos \theta) + [2T_{2+} - T_{2-}] P_2(\cos \theta) + \cdots \right). \]
respectively. The $T_{l\pm}$ are pure isospin-0 partial-wave amplitudes with $J = l \pm 1/2$, and only terms up to $D$ wave are explicitly shown in Eqs. (10). From Table V and Fig. 16, the Legendre fits to the differential cross sections with only four terms are seen to represent the data well. This indicates that the $F$ wave and higher partial waves are very small; they will be neglected in the following discussion. Also, since the $A_4$ coefficient was found to be consistent with zero, and

$$A_4 = \frac{18}{7k^2} [ \{ T_{2+} \}^2 + 4 \text{Re}[T_{2+} T_{2-}] ],$$

this suggests that the $D_{05}$ wave ($T_{2+}$) is quite small. Expressing the $B_1$ in terms of partial waves,

$$B_1 = \frac{2}{k^2} \text{Im} \left\{ T_{0+}[T_{1+}^* - T_{1-}^*] + \frac{1}{5} [9T_{2+}^* T_{1+} - 4T_{2-}^* T_{1+} - 5T_{2-}^* T_{1-}] \right\},$$

$$B_2 = \frac{2}{k^2} \text{Im} \left\{ T_{1+}^* T_{1-} + T_{0+}[T_{2+}^* - T_{2-}^*] + \frac{5}{7} T_{2+}^* T_{2-} \right\},$$

$$B_3 = \frac{2}{5k^2} \text{Im} \left\{ T_{2+}^* T_{1+} + 5T_{2+}^* T_{1-} - 6T_{2-}^* T_{1+} \right\},$$

$$B_4 = \frac{18}{7k^2} \text{Im} \left\{ T_{2+}^* T_{2-} \right\},$$

it would be expected that $B_4$ would also be very small, and Legendre fits with this term give $B_4$ consistent with zero, as expected. From Eqs. (11), the large values of $B_1$ in the momentum range of this experiment are due primarily to the interference of the $S_{01}(T_{0+})$ and $P_{01}(T_{1-})$ partial waves in Gopals’s solution, while the $P_{03}$ and $D_{03}(T_{1+}$ and $T_{2-}$, respectively) waves are smaller in magnitude. Thus, from Eqs. (11), both $B_2$ and $B_3$ would be expected to be much smaller than $B_1$, which is observed.

On the other hand, the Gopal et al. [18] predictions agree only qualitatively with the differential cross section and polarization data, especially at the highest momenta of this experiment, where a rapid energy variation is expected (the beam momentum spread in this experiment must be taken into account when comparing to such partial-wave predictions). Nor do the Legendre coefficients determined from this experiment agree well with the Gopal analysis at all energies. For example, the $B_2$ coefficient would be expected to be noticeably positive due to the interference of the imaginary part of the $S_{01}$ and real part of the $D_{03}$ waves, whereas the data are consistent with zero. Thus, it is expected that the data from this experiment will have a noticeable impact on the $K^- p \rightarrow \Sigma^0 \pi^0$ partial waves.

V. SUMMARY

A set of new differential cross sections and $\Sigma^0$ polarization data for the reaction $K^- p \rightarrow \Sigma^0 \pi^0$ is presented from data collected with the Crystal Ball; see Figs. 16 and 19, and Tables IV and VI. The results are for eight beam momenta between 514 and 750 MeV/c, and nine $\cos(\theta_{c.m.})$ bins. Differential cross sections are generally in good agreement with previous experiments. A peak in data at backward angles is suggestive of a cusp due to the opening of the $K^- p \rightarrow \Lambda \eta$ channel. Statistical uncertainties are usually considerably

FIG. 17. Measured total cross sections for the $K^- p \rightarrow \Sigma^0 \pi^0$ reaction from this experiment (solid circles). Values from Refs. [6–10,13] are shown for comparison. The momenta of several points were offset a few MeV/c to be able to distinguish them. The uncertainties shown are statistical only; systematic uncertainties on the present measurements are approximately $\pm 10\%$.

FIG. 18. Measured differential cross sections for the $K^- p \rightarrow \Sigma^0 \pi^0$ reaction at $\cos(\theta_{c.m.}) = -0.9$ from this experiment (solid circles). The uncertainties shown are statistical only. Values from Armenteros et al. [7] (open circles) and Baxter et al. [10] (open diamonds) are shown for comparison.
smaller than those of the earlier measurements, especially for the $\Sigma^0$ polarization.

Legendre polynomial fits to the data were performed, and the total cross sections derived; see Fig. 17 and Tables V and VII. Agreement with previous experiments is generally good.

Studies using partial-wave fits to previous data suggest that the results in this paper will have the most impact on the $S_{01}$ and $D_{03}$ partial waves and thus on the properties of the $\Lambda(1670)$ and $\Lambda(1690)$ resonances. A new partial-wave analysis will allow insights into quark models and help discriminate between the plethora of models [52].

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