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Analysis of the multireflection effects in compensated liquid crystal devices

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The effect of multiple internal reflections on the extinction ratio of perfectly compensated liquid crystal devices is studied. We find the previously unexplained wavelength dependence of light leakage in an ideally compensated device is caused by the interference of the internally reflected ordinary and extraordinary waves. While these effects have not been previously made clear, they can place a limit on the performance of liquid crystal devices used as displays, optical switches, and optical attenuators. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2464194]

I. INTRODUCTION

Many modes used in liquid crystal devices particularly those with an untwisted structure[1] have residual birefringence in their black state due to the anchoring energy at the alignment layers. For these devices, to obtain a true black state, a passive phase compensator is needed. Ideally, we can compensate the residual birefringence so that the total effective birefringence will be zero at all wavelengths, and we can achieve an extinction ratio that is limited by the polarizer and analyzer used in conjunction with the liquid crystal device.

However, in our calculations and experiments, the black transmittance is much larger than that of the crossed polarizers and depends on the wavelength. We analyzed this situation and found that it is related to multireflected light from internal interfaces within the liquid crystal device. These reflections result in two types of the interference. The first is the interference of the extraordinary ray (*e* ray) and ordinary ray (*o* ray) by themselves and the second one is the interference between *e* and *o* rays. The first type has higher frequency in the wavelength space and is related to the optical path lengths of the *e* and *o* rays independently. The second type has a lower frequency and depends on the residual birefringence of the black state. So, as the residual birefringence increases, there is a greater contribution from the second type of interference. Considering the optical properties of the liquid crystal display, the first type interference could affect the black level almost equivalently for all visible wavelengths, so it does not affect the color of the black state. On the other hand, the second type could cause a color shift because of the lower frequency pattern of the interference in the wavelength space.

The numerical calculation of the light intensity including multireflection effects in a reflection-mode image transducer utilizing a nematic liquid crystal with a 45° twist has been reported.[2] That work covers the liquid crystal thickness dependency of the output intensity with and without isotropic layers for a single wavelength. However, we were not able to find reports concerning the interference effects related to the residual birefringence and their contribution to the optical properties including the wavelength dependence of the extinction ratio.

In this paper, we will consider perfectly compensated liquid crystal devices and show the analytical and numerical calculation results and experimental data showing the effects of the multireflections on the optical properties. Specifically, we will describe the contribution of the residual birefringence to the extinction ratio and its wavelength dependence.

In Sec. II, we will describe the effect of the multireflection analytically in a black state of a simple uniaxial type liquid crystal device that has one pair of interfaces, but this could be expanded to more complicated configuration easily using the same concept. In Secs. III and IV, we will show the calculation results of light transmission when multireflections are considered and analyze the effects. In Sec. V, we will give the experimental data with the numerical calculation including all necessary layers in an example device.

II. CALCULATIONS

In this section, we will consider a simple-uniaxial-type liquid crystal device with its black state compensated perfectly using passive optical retarders. Light is assumed to be incident at the normal direction and is reflected only at the one pair of surfaces; on either side of the liquid crystal layer in air. [In actual liquid crystal devices, there are no air layers inside the devices. However, in an actual liquid crystal display (LCD) there can be reflections from the multiple interfaces. Those reflections can provide a similar (but perhaps smaller) effect as will be shown in Sec. V.] Figure 1 shows this configuration with two crossed polarizers, and in this figure, the δ and Γ are the phase changes of the light at the liquid crystal and compensator, respectively. The electric field after the polarizer is expressed as

$$\mathbf{E} = \hat{x}\frac{1}{\sqrt{2}}E_{\text{in}}\exp[i(kz - wt)]. \tag{1}$$

This electric field that has wavelength λ in free space can be split into two eigenmodes, extraordinary mode (*e* mode) and ordinary mode (*o* mode), in the liquid crystal layer, and they will propagate with different phases. Each wave is also mul-

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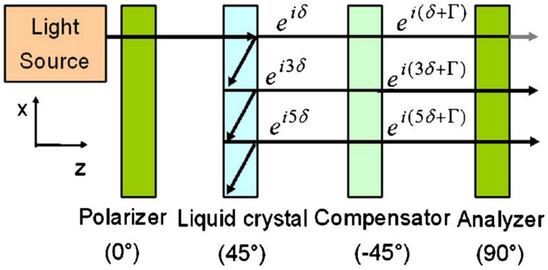

FIG. 1. (Color online) Simple layout of a liquid crystal device. The compensator is designed to compensate the phase of the black state perfectly in the visible wavelength range.

tireflected at the liquid crystal-air interface and has the following form after the liquid crystal layer (time related terms have been dropped here for the simplicity):

$$\mathbf{E}_e = \hat{e}\frac{1}{2}E_{\text{in}}t^e_{\text{ap}}[\exp(i\delta_e) + r_e^2\exp(i3\delta_e) + r_e^4\exp(i5\delta_e) + \ldots]t^e_{\text{pa}} \tag{2}$$

$$\equiv \hat{e}E_e\frac{(1-r_e^2)\exp(i\delta_e)}{1-r_e^2\exp(i2\delta_e)}, \tag{3}$$

$$\mathbf{E}_o = -\hat{o}E_o\frac{(1-r_o^2)\exp(i\delta_o)}{1-r_o^2\exp(i2\delta_o)}, \tag{4}$$

where $\hat{e}$ and $\hat{o}$ are the unit vectors along the electric field directions of the extraordinary ray (e ray) and ordinary ray (o ray), respectively. The t^e_{ap} and t^e_{pa} are the transmission coefficients of the e mode at the interface, air-liquid crystal, and r_e and r_o are the reflectance coefficients of the e and o modes, respectively, at the same interface. The δ_e and δ_o are the spatial phase changes of the e and o modes at the liquid crystal layer. Those parameters are expressed using the refractive indices (n_e, n_o) and the thickness (d) of the liquid crystal layer,

$$r_e^2 = \left(\frac{n_e - n_{\text{air}}}{n_e + n_{\text{air}}}\right)^2, \tag{5}$$

$$r_o^2 = \left(\frac{n_o - n_{\text{air}}}{n_o + n_{\text{air}}}\right)^2, \tag{6}$$

$$\delta_e = \frac{2\pi n_e d}{\lambda}, \tag{7}$$

$$\delta_o = \frac{2\pi n_o d}{\lambda}. \tag{8}$$

After the liquid crystal layer, the e and o rays enter the compensator (refractive indices n_e' and n_o', thickness d'), and will have the phase change Γ_e and Γ_o, respectively. The electric fields after the compensator are

$$\mathbf{E}_e' = \hat{e}E_e\frac{(1-r_e^2)}{1-r_e^2\exp(i2\delta_e)}\exp[i(\delta_e+\Gamma_o)], \tag{9}$$

$$\mathbf{E}_o' = -\hat{o}E_o\frac{(1-r_o^2)}{1-r_o^2\exp(i2\delta_o)}\exp[i(\delta_o+\Gamma_e)], \tag{10}$$

where

$$\Gamma_e = \frac{2\pi n_e' d'}{\lambda}, \tag{11}$$

$$\Gamma_o = \frac{2\pi n_o' d'}{\lambda}. \tag{12}$$

After the analyzer, the total electric field and transmittance can be calculated as follows:

$$\mathbf{E}_{\text{out}} = \hat{y}\frac{1}{\sqrt{2}}(E_e' + E_o'), \tag{13}$$

$$T \equiv T_e + T_o + T_{eo}, \tag{14}$$

where

$$T_e = \frac{1}{2}E_e^2\frac{(1-r_e^2)^2}{1+r_e^4-2r_e^2\cos(2\delta_e)}, \tag{15}$$

$$T_o = \frac{1}{2}E_o^2\frac{(1-r_o^2)^2}{1+r_o^4-2r_o^2\cos(2\delta_o)}, \tag{16}$$

$$T_{eo} = -\frac{1}{2}E_eE_o(1-r_e^2)(1-r_o^2) \times \left\{\frac{1}{[1-r_e^2\exp(i2\delta_e)][1-r_o^2\exp(-i2\delta_o)]} + \frac{1}{[1-r_e^2\exp(-i2\delta_e)][1-r_o^2\exp(i2\delta_o)]}\right\}, \tag{17}$$

where we used the fact that the difference of the phase between e and o modes at the liquid crystal layer is same with that of the compensator layer because we assume the black state of the LCD is compensated perfectly.

Equations (14)–(17) show that there are three contributions to the total transmittance in a LCD compensation system; T_e is the transmittance related to the interference of the pure e mode by itself due to the multireflection in a LCD, and T_o is the transmittance that came from the interference of the pure o mode with the same reason. The last term T_{eo} is a coupled term between the e and the o modes. So, T_e and T_o depend on the absolute light path lengths (n_ed/λ, n_od/λ) of the e and the o modes, respectively, but the T_{eo} is related to the relative difference of the light path length ($|n_ed/\lambda - n_od/\lambda|$) between the e and the o modes.

III. A LIQUID CRYSTAL DEVICE WITH NO RESIDUAL RETARDATION IN THE LOW LIGHT TRANSMISSION STATE

In an ideal electrically controlled birefringence (ECB)-type LCD, the residual birefringence at the high voltage (black state) is very low, because most of the liquid crystal molecules align along the external electric field. However, the molecules near the surfaces are not completely aligned due to the anchoring energy at the interface between the liquid crystal and the alignment layer. We assume here that there is no residual birefringence at the high voltage for simplicity, but still the device has different reflectance coefficients (r_e, r_o) of the e and o modes at the interfaces,

$$\delta_e = \delta_o \equiv \delta, \quad \Gamma_e = \Gamma_o = 0, \quad r_e \neq r_o.$$

Then, the electric fields just before the analyzer, Eqs. (9) and (10), are simplified to be

$$\mathbf{E}'_e = \hat{e} E_e \frac{(1 - r_e^2)}{1 - r_e^2 \exp(i2\delta)} \exp(i\delta), \tag{18}$$

$$\mathbf{E}'_o = - \hat{o} E_o \frac{(1 - r_o^2)}{1 - r_o^2 \exp(i2\delta)} \exp(i\delta). \tag{19}$$

In these equations, $\mathbf{E}'_e$ and $\mathbf{E}'_o$ are in the same phase but their amplitudes depend on the light wavelength. Therefore, the direction of the total field (vector summation of these two E fields) varies from the absorption axis of the analyzer periodically with the same phase of the $\mathbf{E}'_e$ and $\mathbf{E}'_o$ in the wavelength space, and the angle variation $[\theta(\lambda)]$ between them is calculated from the formulas (18) and (19),

$$\theta(\lambda) = \tan^{-1}\left(\frac{1 - r_e^2}{1 - r_o^2} \frac{1 - r_o^2 \exp(i2\delta)}{1 - r_e^2 \exp(i2\delta)} \right) - \frac{\pi}{4}. \tag{20}$$

When the direction of the total field is in the absorption axis of the analyzer ($\theta=0$), it gives zero transmittance, and as the angle (θ) increases, the light leakage becomes bigger. The amplitude of this angle variation is not only proportional to the relative difference of the reflective coefficients ($|r_e - r_o|$), but also depends on the absolute value of each of them. Therefore, from Eqs. (5), (6), and (20), the light leakage of a black state LCD becomes larger as the refractive indices mismatching and the relative difference of the refractive indices ($|n_e - n_o|$) of the e and o modes increase at the interfaces. On the other hand, the amplitude of the $\theta(\lambda)$ is independent of the wavelength if there is no dispersion of the refractive indices, so the amplitude of light leakage of a black state are same in whole wavelength region, and it does not affect the black color.

Figure 2 shows the calculation results of the separate transmittance (T_e, T_o, T_{eo}) and the total transmittance T with the angle variation (θ). As we mentioned above, the e and o modes are in phase, and the valley and ridge points of the total transmittance (T) are constant as the wavelength changes. Also, the angle variation (θ) is in the same phase with that of the total transmittance exactly. Therefore, the minimum transmittance at the black state is limited by the amplitude of the interference pattern (T), and the black color is hardly affected in this type of interference.

IV. A LIQUID CRYSTAL DEVICE WITH RESIDUAL RETARDATION IN THE LOW LIGHT TRANSMISSION STATE

As an example mode, we will consider a pi-cell[3,4] type LCD. In this mode, as in others, there is a residual birefringence at black voltage that requires the use of a passive optical retardation film to compensate it. Figure 3 shows the effective birefringence change at normal direction for the incident light wavelengths, 450, 550, and 650 nm, in a pi cell [liquid crystal: LC01 (n_e=1.6971, 1.6644, 1.6457, n_o=1.5277, 1.5070, 1.4951 at 450, 550, and 650 nm, respectively; $\Delta\varepsilon$=9.4); cell thickness of 5.53 μm]. As in this figure, the pi cell still has ~100 nm birefringence at around the black voltage (5~6 V). Therefore, we have these relationships in a pi cell,

$$\delta_e \neq \delta_o, \quad \Gamma_e \neq \Gamma_o, \quad r_e \neq r_o.$$

So, from Eqs. (9) and (10), we know that the e-mode and o-mode components of the electric fields $(\mathbf{E}'_e, \mathbf{E}'_o)$ after the compensator are out of phase in time and wavelength space. Therefore, the light after compensator is elliptically polarized, and the ellipticity and the angle of the major axis are functions of the light wavelength. This causes the total transmittance to have a beat frequency phenomenon after the analyzer.

Figure 4 shows the calculation results of the transmittance (T_e, T_o, T_{eo}) contributed from each mode, and total

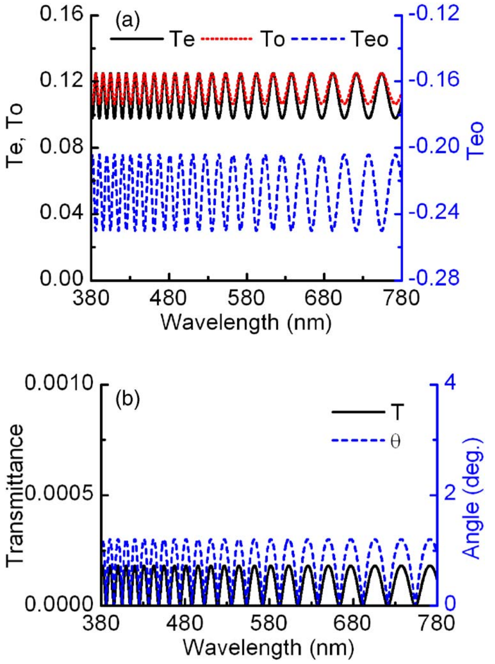

FIG. 2. (Color online) (a) Transmittances of the each mode, (b) total transmittance with the angle variation in the black state of an ECB type LCD. T_e and T_o are in phase, but have different amplitudes, and the minimum and maximum values of the total transmittance are independent of the wavelength. (Δnd=0 nm, d=5.53 μm, n_e=1.656, and n_o=1.5 at the surface).

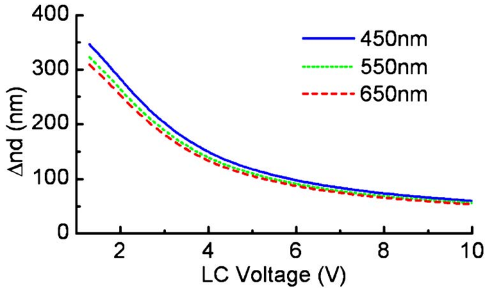

FIG. 3. (Color online) Calculation results of the effective birefringence at normal direction in a pi cell.

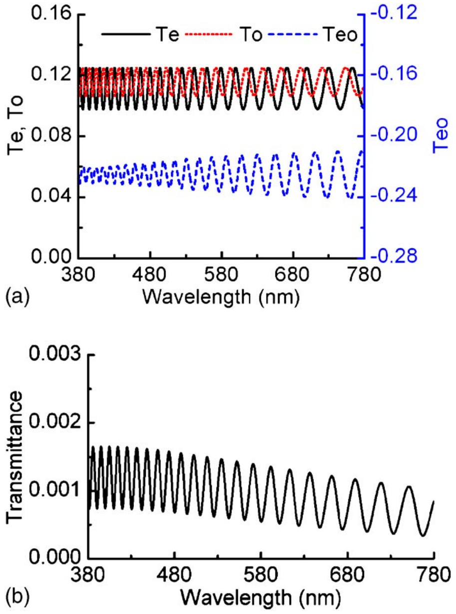

FIG. 4. (Color online) (a) Transmittances of each mode, (b) total transmittance in the black state of a pi-cell-type LCD. T_e and T_o are not in phase and have different amplitudes. The total transmittance has two frequency modes, and the minimum points of the transmittance are different for each wavelength. ($n_e=1.656$ and $n_o=1.5$ at the surface).

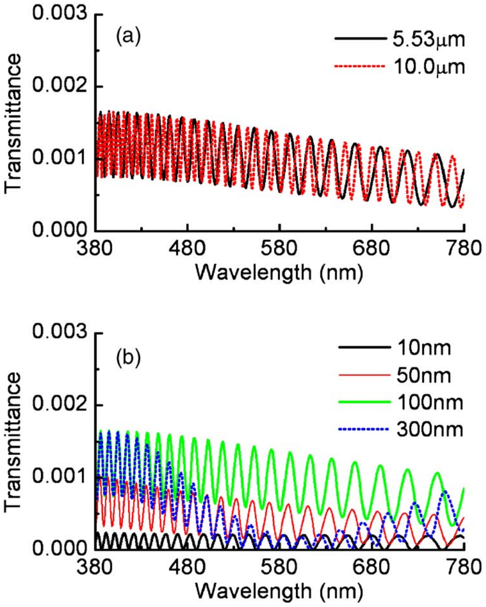

FIG. 5. (Color online) (a) Thickness effect ($\Delta nd=100$ nm fixed) and (b) phase difference effect ($d=5.53$ μm fixed) on the total transmittance of a pi-cell-type LCD in the black state. Light path lengths of the *e* and *o* modes affect the high frequency mode, and the low frequency mode is related with the phase difference between the *e* and *o* modes.

transmittance T at the black state in a pi cell, respectively. The magnitude of the residual birefringence we used here is 100 nm, and the compensator compensates the value exactly. The cell thickness in this calculation is 5.53 μm again. As is seen in Fig. 4(a), T_e and T_o are not in phase and have different amplitudes which depend on the reflective coefficients (r_e, r_o). In Fig. 4(b), the minimum values (valley points) of the transmittance curve are not zero, and their values vary as the wavelength changes (unlike the ideal ECB-type LCD). These phenomena make the black level higher, and not only limit the contrast ratio the display can reach (here 500–1000:1) but also lead to the color shift of a black state of a LCD.

The total transmittance [Fig. 4(b)] of a "pi-cell-type LCD" has two frequency modes, roughly, high frequency and low frequency. In order to analyze the source of the modes, we calculated the total transmittance at the two conditions; first, the same phase difference ($\Delta nd=100$ nm) but different cell thicknesses ($d=5.53, 10$ μm) shown at Fig. 5(a) and second, the same cell thickness ($d=5.53$ μm) for several phase differences ($\Delta nd=10, 50, 100, 300$ nm) shown at Fig. 5(b). From these results, we know that the higher frequency mode is coming from the interference of the *e* and *o* modes independently in a LCD and depends on the absolute light path lengths of the each mode. The lower frequency mode is caused by the interference between *e* and *o* modes and affected by the relative phase difference of them. Considering the optical properties, the lower frequency mode is much more critical, so we need to reduce the residual birefringence to improve the black quality of a LCD irrespective of the compensation films. Another important thing is that the lower frequency mode is governed by the optically anisotropic layers, so at which interface the multireflection takes place is not so important in a real LCD because other layers except the liquid crystal are mostly isotropic material. Therefore, the spacing between the interfaces causing the reflections affects only on the higher frequency mode, as in Fig. 5(a) and does not effect the color shift.

V. EXPERIMENTAL RESULTS

To confirm the calculation results, we did the experiments using a pi cell (liquid crystal: LC01; cell thickness of 5.53 μm). We used commercialized uniaxial compensator to compensate phase retardation in the pi cell (normal direction) in black state (5.15 V). The compensated and measured wavelength range was 400–700 nm, and the step was 10 nm. However, the phase difference of the LCD is hard to compensate perfectly in all of the wavelength range simultaneously. Therefore, for each wavelength, we changed the retardation values of the compensator to have minimum transmittance (the phase difference of the pi cell is compensated exactly). Figure 6 is the measurement layout. We used crystal polarizers to achieve high extinction ratio in this experiment (Melles Griot, model 03PTO003/A, extinction ratio <1/100 000). Each optical component in the setup is separated physically, and the surfaces of the crystal polarizers and the compensator are treated for antireflection in visible light

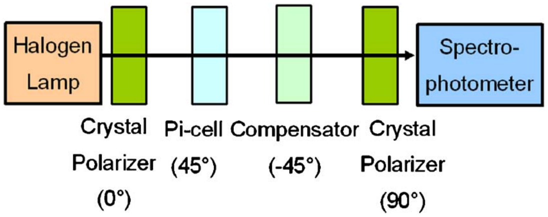

FIG. 6. (Color online) Measurement setup of a pi-cell compensation system. LC volt: 5.15 V (black state).

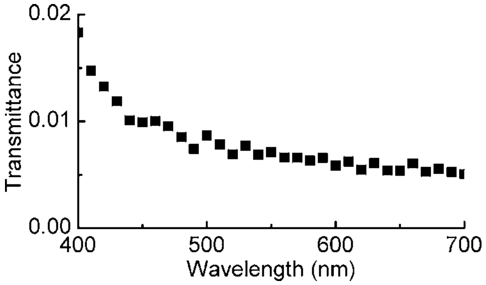

FIG. 7. Transmittance of the pi cell compensated with the compensator at each wavelength separately. Reference ($T=1$): parallel crystal polarizers; LC volt: 5.15 V (black state).

wavelength range. Therefore, the main sources of the multiple reflections are the either sides of the pi cell and the layers inside the pi cell such as substrates, electrodes, alignment layers, and liquid crystal.

Figure 7 shows the minimum transmittance we take from the experimental raw data. As we can see in Fig. 7, there is a big light leakage in all of the wavelengths, and especially in the blue region. This means the maximum contrast ratio of the pi cell is limited by the multireflection effect, and also that effect could cause the blueshift in the color of the black state. In case of this experiment, the calculation results of the change of the color coordinates [1931 CIE (International Commission on Illumination)] for the standard D65 light source are

$$x=0.3127, \quad y=3291 \rightarrow x=0.2722, \quad y=0.2802.$$

We also acquired data with a slightly different experimental setup (Fig. 8). In this case, we used commercialized hybrid aligned discotic-negative films designed for the pi cell as a compensator and sheet polarizers in place of the crystal polarizers. All films and the pi cell are combined as a one unit so that we can achieve minimum indices mismatching at the interfaces unlike the previous experiment. We applied the voltage range from 5.0 to 5.6 V to liquid crystal layer to get the minimum transmittance at each wavelength where the phase of the liquid crystal is compensated by the films perfectly because the pi cell and the films are acting like the crossed uniaxial layers. From this experiment, we see that [Fig. 9(a)] there is a big multireflection effect in the black state of the real pi cell, and its effect is much more severe in the blue region. An important fact to note is that the minimum transmittances we achieved at each wavelength are bigger than the transmittance of the crossed polarizers ($T=0.0003-0.0004$ at 550 nm wavelength), and that difference increases in the blue region even though we compensated the phase of the liquid crystal separately in each wavelength. Therefore, this situation should lead to the blueshift of the black color of the pi cell, and the maximum contrast ratio we can get is around 1000.

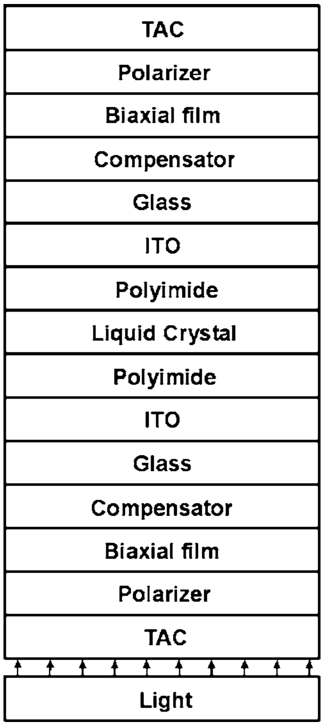

FIG. 8. Optical stack of the real pi cell. All layers are combined as a single unit without any air gaps.

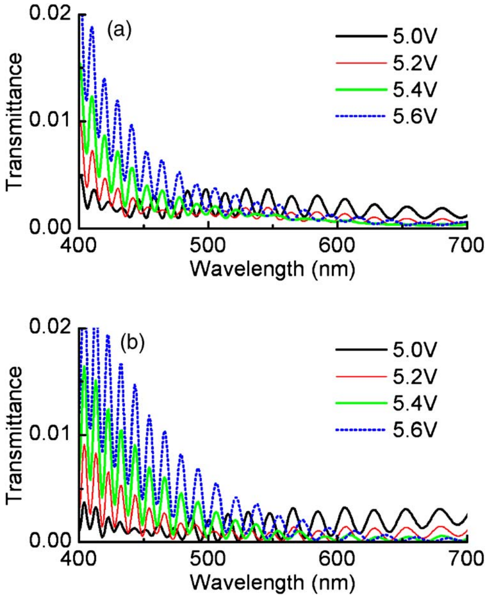

FIG. 9. (Color online) (a) Measurement results and (b) numerical calculation results of the transmittance of the pi cell compensated with the hybrid aligned discotic-negative films for different applied voltages. Reference ($T=1$): parallel sheet polarizers.

We simulated this experiment by using a numerical relaxation technique to calculate the director field in the pi cell and by using the Bereman 4×4 method[5–7] to calculate the optical properties. Figure 9(b) shows the results. In these calculations, we used exactly the same optical stack as Fig. 8 and considered the dispersion of the refractive indices of all layers. The optical parameters of layers that we did not have from the manufacturer, such as the polarizer, glass, Indium Tin Oxide (ITO), and polyimide, were measured by ourselves at a resolution of 1 nm. The only measured parameter that was adjusted to acquire Fig. 9(b) was the thickness of the liquid crystal layer. The thickness of the liquid crystal layer was measured before filling the liquid crystal as 5.53 μm, but we adjusted the value to 5.18 μm during the calculation to achieve the best fitting to the experimental data. We think this is acceptable because the thickness of the liquid crystal layer could be changed during the cell-making process.

VI. CONCLUSION

We calculated the multireflection effects analytically and numerically in the black state of a compensated liquid crystal device and compared the results with the measured transmittance of an example device.

According to our analysis, there are two types of interference in devices with significant residual retardation in the low transmission state that is compensated by a passive optical retarder. The first one is due to the pure *e* ray and pure *o* ray by themselves, and the second one is coming from the coupling between the *e* and *o* modes. The first type has higher frequency in the wavelength space and is related to the optical path length of the *e* and *o* rays and is independent of their difference. Most of the modes used in liquid crystal devices have this type of interference. The second type of interference has lower frequency than that of the first one and depends on the residual birefringence of black state. So, as the residual birefringence increases, the second type of interference becomes more significant. In the viewpoint of the optical properties of a liquid crystal device, the first type of interference could affect the black level and extinction ratio almost equivalently for visible wavelength region, while the second type could cause a wavelength dependence of the extinction ratio, or a color shift of the black state because of the lower frequency pattern of the interference in the wavelength space.

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