A Study of the Reduction of Excessive Energy Generated by Strong Winds on Power Lines via a Velocity Damping Controller at the Transmission Tower

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Abstract: In this research, we are pursuing the robustness of a self-excited vibrational system with negative damping. In practice, this is manifested as conductor galloping of overhead power lines, which is an oscillation of the lines caused by strong winds. Improved transmission tower designs are needed which are capable of combating excessive stresses exerted on the tower by the galloping power lines. Our model of this self-excited system shows that the oscillations can be controlled by adding a boundary velocity feedback controller at the transmission tower. Using the Adomian decomposition method, we show there is a closed form analytical solution which can predict the stabilizability of the system under certain conditions, and a numerical computation is performed to demonstrate the theoretical results of this article. Through this research, power transmission systems that are more reliable and resistant to galloping can be engineered.

Adomian Decomposition Method (continued)

Apply the integral operator:

\[
L_{\nu}^2 u{\nu} = \frac{d^2 u_{\nu}}{dt^2} + \frac{c^2}{\mu} u_{\nu} = 0
\]

Assume \( u(x,t) = f(x) \), the series is an exact analytical solution for the self-excited system is stabilizable.

\[
\begin{align*}
L_{\nu}^2 u_{\nu} &= \frac{d^2 u_{\nu}}{dt^2} + \frac{c^2}{\mu} u_{\nu} = 0 \\
\sum_{n=0}^{\infty} u_{\nu}(x,t) &= f(x) \\
L_{\nu}^2 u_{\nu}(x,t) &= f(x) - L_{\nu}^2 u_{\nu}(x,t) = 0
\end{align*}
\]

Initial Conditions
\[
\begin{align*}
u(x,0) &= f(x) \\
u_t(x,0) &= 0
\end{align*}
\]

Mixed Boundary Conditions
\[
\begin{align*}
u(0,t) &= 0 \\
u(L,t) &= 0
\end{align*}
\]

Energy of the System:
\[
E(t) = \frac{1}{2} \int_0^L \left( u_t^2 + c^2 u_{xx}^2 \right) dt
\]

Convergence of the Series
When \( \lim_{n \to \infty} \sum_{n=0}^{\infty} u(t) \), exists the series is an exact analytical solution for \( u(x,t) \). The decomposition method has been shown to be convergent in many cases [1], [2], [8].

Adomian Decomposition Method
The Operator Form is: \( Lu + R u + Nu = g(x,t) \)
- \( L \) is a linear operator
- \( R \) is also a linear operator
- \( N \) is a nonlinear operator
\( L_{\nu}^2 \) is an integral operator \( \Rightarrow L_{\nu}^2 \equiv \int (-\nu, \cdot, \nu) dt \)

\[
\begin{align*}
Lu &= u_t \\
R u &= -c^2 u_{xx} - \mu u_t \\
g(x,t) &= g(x,t) \\
N u &= 0
\end{align*}
\]

The governing equation is: \( Lu + R u = 0 \)

Closed Form Analytical Solution
\[
\begin{align*}
u(x,t) &= \exp \left( \frac{\nu}{2} \right) \left( \cos(\omega t) - \frac{\mu \sin(\omega t)}{2\omega} \right)
\end{align*}
\]

Aftertreatment Technique
Let the Laplace and inverse Laplace transforms be denoted, respectively, \( L \) and \( L^{-1} \) and the Padé’ approximant of order \( [j,k] \) be denoted \( P[j,k] \). Then, the aftertreatment technique is the following:

\[
\begin{align*}
u(x,t) &= L^{-1} P[j,k] \sum_{n=0}^{\infty} u_n \\
Closed \ Form \ Analytical \ Solution
\end{align*}
\]

Statements and Proofs

Proposition: The system: \( \mu u_t - c^2 u_{xx} = 0 \) for \( x, t > 0 \) with initial and mixed boundary value conditions which are the same as for the self-excited system is stabilizable.

Proof: See [7] for the proof of this proposition.

Lemma 1: There exists \( \tau \) such that the self-excited velocity damping coefficient \( \mu \ll 1 \), which forces \( \tau \) to become zero, whenever \( \tau \gg 0 \).

Proof: From (2), (3) and (4): \( \mu = \frac{\omega}{\tan(\omega)} \). Then, \( \mu = \frac{\omega}{\tan(\omega)} < 1 \) for \( \theta \in (0,\pi) \) and \( \mu < \frac{\omega}{\tau} \) for \( \tau \) increases. Such that \( \mu \) becomes much less than 1.

Lemma 2: The energy of the self-excited system, \( E(t) \), decays to zero asymptotically as \( \tau \) increases.

Proof: Differentiate \( E(t) \) to obtain \( \dot{E}(t) = \mu \frac{1}{2} \int_0^L |u_t|^2 + c^2 u_{xx}^2 dt \). Then use (1) to replace \( \mu u_t \). Next apply integration by parts. This gives \( \dot{E}(t) = \mu \frac{1}{2} \int_0^L |u_t|^2 dt - \mu u_t(0,1) + c^2 u_{xx}(x) \). Then, applying (2) and (3) leads to the conclusion that the energy decays asymptotically to zero as \( \tau \) increases.

Theorem: The self-excited system is asymptotically stable provided that \( \mu < 1 \), i.e., \( \lim_{\tau \to \infty} E(\tau) = 0 \).

Proof: The theorem follows immediately from the Proposition and the Lemmas.

Numerical Simulation
The figure on the left confirms the theoretical results by showing that the energy decreases asymptotically for \( \mu < 1 \).

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References