Universal Off-Axis Light Transmission Properties of the Bright State in Perfectly Compensated Liquid Crystal Devices

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Universal off-axis light transmission properties of the bright state in perfectly compensated liquid crystal devices

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We studied the off-axis light transmission characteristics of the bright state of common liquid crystal device modes. The dark state of these device modes is optically compensated to have minimum light transmittance at all angles. Our research shows there is an unexpected universal shape of the off-axis light transmission value in its bright state, regardless of what liquid crystal mode is used. To understand this surprising fact, we consider simple dark and bright state models in terms of phase retardation and transmittance. © 2007 American Institute of Physics. [DOI: 10.1063/1.2751113]

I. INTRODUCTION

The viewing angle dependence of light transmittance in liquid crystal devices is a well-known feature and one of the biggest problems. The dark (black) state is especially important because it is critical to the contrast ratio of the device. On this account, a great deal of research has been done to reduce the dark state transmittance and its variation at all viewing angles. On the other hand, the bright (white) state determines the luminance and the spectral variations of the transmittance as a function of the angle of the incident light. Therefore, this should be another vital factor when deciding the optical properties of the devices. However, we have not seen those studies dealing with the off-axis light transmission properties (viewing angle properties) of the bright state of general liquid crystal devices.

In this article, we investigate the off-axis light transmission properties of the bright state in the most common liquid crystal devices whose dark states are optically compensated perfectly. According to our results, there is an interesting universality in the off-axis light transmission properties of the bright state of liquid crystal devices, which is independent of the display modes. In order to explain these interesting facts, we make simple dark and bright state models that can be applied to general liquid crystal devices and analyze them in terms of the effective retardation and transmittance.

In Sec. II, we will show the off-axis light transmission properties of the bright state in the most commonly used liquid crystal devices. In Sec. III, we will introduce simple dark and bright state models, calculate the angular dependencies of the effective retardation and transmittance of the models, and then compare the results with those of Sec. II. Finally, we will give our conclusions in Sec. IV.

II. OPTICAL PROPERTIES OF GENERAL LIQUID CRYSTAL DEVICES

Figure 1 shows the numerical calculation results of the off-axis light transmission properties of the bright states (incident light wavelength: 550 nm) in the most common liquid crystal device modes such as the electrically controlled birefringence (ECB), vertical alignment (VA), twisted nematic (TN), Pi cell, and symmetric splay cell. During the calculation, we used the numerical relaxation technique to get the director field of the bright state in the liquid crystal layer. We also used the $2 \times 2$-matrix method to calculate the optical properties of the devices. The dark state of each mode is optically compensated via two (above and below the liquid crystal layer) compensators with the hybrid-negative $C$ structure. They have exactly the same angular distribution as the directors of the liquid crystal layer of the dark state and their extraordinary, ordinary refractive indices are the same as the ordinary, extraordinary indices of the liquid crystal, respectively. This is in accordance with Mori's argument that the effect of each layer of a positive birefringence material in a liquid crystal device can be optically compensated by a layer of negative birefringence materials with the same optic axis orientation. Other methods, for example, using positive $O$ plates or biaxial material can be used for optical compensation of the dark state. However, their final destinations are the same, i.e., we want to make the total effective retardation of liquid crystal and compensator layers zero in all viewing

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{(Color online) Numerical calculation results of the bright state off-axis light transmission properties of the common liquid crystal devices (a) in the director plane and (b) out of the director plane.}
\end{figure}

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directions and wavelengths if we use ideal polarizers. Therefore, the more we perfectly compensate the dark state, the more the final compensator effects are the same, no matter what compensation schemes are used.

The thicknesses of the liquid crystal layer and the compensators of each display mode are determined by meeting two conditions. First, it gives the minimum transmittance at all viewing angles in the dark state. Second, the transmittance of the bright state at the normal direction has a specific value ($T_w$). This value can be chosen from zero to maximum transmittance that we can achieve under the crossed polarizers, but a bit smaller than the maximum value is usually used to achieve high transmittance and to escape color shift in the off-axis viewing directions. In this article, we set the value to $3/4$ of the maximum transmittance because this is a similar condition in real liquid crystal devices, and this value is not a critical factor in determining the physical concept of devices. The detailed specifications and layout are in Table I and Fig. 2. We want to make clear that the specific numbers are only for the purposes of producing a graph that demonstrates the features of the general concepts considered here.

From these transmittance figures, we notice that even though the director configurations and effective birefringence of the liquid crystal layers are completely different from each other, as in Fig. 3, the off-axis light transmission properties of the compensated liquid crystal devices, amazingly, have unified shapes. (We cannot define the birefringence out of the director plane because the optic axes of the directors are apparently twisted, so we calculated it only in the director plane. For the same reason, only the off-axis light transmission properties are calculated in TN mode.) In the viewing angle of the director plane [Fig. 1(a)], all the transmittance curves have similar “bell” shapes, i.e., the transmittance constantly decreases as the viewing angle increases from the normal direction no matter what liquid crystal modes are used. On the other hand, the transmittance out of the director plane [Fig. 1(b)] rises as the viewing angle increases from the normal direction and then falls after passing the specific angles (about ±50° in these calculations), regardless of the director configurations of the liquid crystal in their bright states. These results show that the off-axis light transmission properties of the bright state in single domain liquid crystal devices have a common shape, independent of their modes, as long as their dark states are optically compensated to give the lowest transmittance for all viewing angles. This surprising fact motivated us to investigate the reason for this in the next section.

### III. UNIVERSAL SIMPLE MODEL

#### A. Dark and bright state modeling

Let us consider the liquid crystal director configurations of several nontwisted common liquid crystal devices (LCDs) such as ECB, VA, Pi cell, and symmetric splay cell. Figure 4(a) shows cartoons of the liquid crystal director configurations of each device. Each of the devices has two states, state 1 and state 2, depending on applied voltage, and either of them is bright state and the other is dark state. In these two states, we can notice that there are simply two parts in a liquid crystal layer: the static part (empty director shape in the figure) and dynamic part (filled director shape). In the static part, most of the liquid crystal directors only slightly change their orientation between the two states. On the contrary, the directors in the dynamic part are very sensitive to the applied voltage, so the orientation of the directors is com-

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**TABLE I.** Cell parameters of the common liquid crystal (LC) devices we used. These specific numbers are only for the purposes of producing a graph that demonstrates the features. (LC: $n_1 = 1.6644$ and $n_2 = 1.5070$ at $\lambda = 550$ nm; $\Delta n = 9.4$ for VA, +9.4 for other devices).

<table>
<thead>
<tr>
<th>Devices</th>
<th>Thickness (μm)</th>
<th>Voltage (V)</th>
<th>Pretilt angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LC Compensator</td>
<td>Bright</td>
<td>Dark</td>
</tr>
<tr>
<td>Pi cell</td>
<td>5.037</td>
<td>2.255</td>
<td>5.5</td>
</tr>
<tr>
<td>ECB</td>
<td>1.423</td>
<td>0.637</td>
<td>5.5</td>
</tr>
<tr>
<td>VA</td>
<td>1.553</td>
<td>0.541</td>
<td>0.0</td>
</tr>
<tr>
<td>TN</td>
<td>2.038</td>
<td>0.832</td>
<td>0.0</td>
</tr>
<tr>
<td>Splay cell</td>
<td>1.87</td>
<td>0.934</td>
<td>5.0</td>
</tr>
</tbody>
</table>

---

**FIG. 2.** Layout of the common liquid crystal devices.
completely different between the state 1 and state 2 of each device. Considering the director-tilt angles, both states have vertical and horizontal components, but state 1 has a larger vertical component than state 2, and state 2 has a larger horizontal component than state 1. Therefore, the only difference between the two states is in the dynamic layers, but the bright states have a net birefringence that is a function of the viewing direction.

B. Calculations

The light transmittance ($T$) of the simple model [Fig. 4(c)] under ideal crossed polarizers can be written as follows:

$$T = \frac{1}{2} T_{ap} T_{pa} \sin^2 \frac{\Gamma}{2} = T_{\text{max}} \sin^2 \frac{\Gamma}{2},$$

where $T_{ap}$ and $T_{pa}$ are the transmittances with considering the surface reflections at the interfaces between air and polarizers in the incident and exit media, respectively. They can be calculated through the Fresnel equations:

$$T_{ij} = \frac{n_i \cos \theta_i}{n_j \cos \theta_j} [t_{ij}]^2,$$

where $n_i$ and $n_j$ are the refractive indices of the incident and refracted media, $\theta_i$ and $\theta_j$ are the incident and refraction angles, respectively, and $t_{ij}$ is the transmission coefficient. The $p$-polarization and $s$-polarization components of $t_{ij}$ are expressed in Eqs. (3) and (4),

$$t_{ij}^p = \frac{2n_i \cos \theta_i}{n_j \cos \theta_i + n_i \cos \theta_j},$$

$$t_{ij}^s = \frac{2n_i \cos \theta_i}{n_j \cos \theta_i + n_j \cos \theta_j}.$$

The $\Gamma$ in Eq. (1) is the phase difference between $e$ ray and $o$ ray in the anisotropic layers and written in terms of incident light wavelength ($\lambda$) and effective retardation ($\Delta n d_{\text{eff}}$) of media when we assume the $e$ ray and $o$ ray have the same path in the media:

$$\Gamma = \frac{2\pi}{\lambda} \Delta n d_{\text{eff}}.$$

[The exact expression of the phase difference ($\Gamma$) is $\Gamma = (K_{oc} - K_{oe}) d$, where $K_{oc}$ and $K_{oe}$ are $\gamma$ components of the wave vectors of the $e$ ray and $o$ ray, respectively. For simplicity, we assume the $e$ ray and $o$ ray have the same path in this article, which is reasonable in a real situation.] In the normal direction, let us say the transmittance of the bright state is $T_o$, and the corresponding phase difference is $\Gamma_o$. Then, as indicated by Eq. (1),

$$\Gamma_o = 2 \sin^{-1} \left( \frac{T_o}{T_{\text{max}}} \right)^{1/2}.$$
This phase difference is purely caused by the positive A plate (bottom layer) of the bright state model in mode 1 of Fig. 4(c) because the optical axis of the negative C plate (top layer) is in the normal direction. With similar reasoning, the phase difference is exclusively related to the negative A plate (top layer) of the bright state model in mode 2. From these facts and Eqs. (5) and (6), we can determine the thicknesses of the positive A(C) plate (bottom layer) in mode 1 and the negative A plate (top layer) in mode 2 that gives the bright state transmittance \( T\) for a given material in the normal direction.

In mode 1, the incident light angle dependence of the effective retardation (\( \Delta n d_{\text{eff}} \)) of the positive A(C) plate (bottom layer) and negative C plate (top layer) are calculated in Eqs. (7) and (8), respectively,

\[
\Delta n d_{\text{eff}}^{\text{positive}} = \left( \frac{n_e n_o}{\sqrt{n_e^2 \cos^2 \psi + n_o^2 \sin^2 \psi}} - n_o \right) \frac{\Gamma o \lambda}{2 \pi \Delta n \cos \alpha},
\]

\[
\Delta n d_{\text{eff}}^{\text{negative}} = \left( \frac{n_e' n_o'}{\sqrt{n_e'^2 \cos^2 \psi' + n_o'^2 \sin^2 \psi'}} - n_o' \right) \frac{d'}{\cos \alpha'},
\]

where \( n_e(n_o) \) and \( n_e'(n_o') \) are the refractive indices of the e ray and o ray, \( \phi(\psi') \) is the angle between the optic axis of the director and light propagation vector, \( \alpha(\alpha') \) is the light incident polar angle in the medium, and \( d' \) is the thickness of the negative C plate. In a given assumption that the transmittance of the dark state should be minimized, the total retardation of the dark state model (\( \Delta_{\text{dark}} \)) should be zero at a given light incident angle

\[
\Delta_{\text{dark}} = \Delta n d_{\text{eff}}^{\text{positive}}|_{\text{dark}} + \Delta n d_{\text{eff}}^{\text{negative}}|_{\text{dark}} = 0.
\]

From Eqs. (7)–(9), the thickness of the negative C plate \( (d') \) of mode 1 can be decided. In the same way, we can also determine the thickness of the positive A(C) plate (bottom layer) in mode 2.

C. Analyses of the effective retardation

In this section, we are going to calculate the total effective retardation of the bright state by way of the formula in the previous section in the two viewing planes: director plane \([x−z\text{ plane in Fig. } 4(c)]\) and out of the director plane \([y−z\text{ plane in Fig. } 4(c)]\). The extraordinary and ordinary refractive indices of the positive (negative) plates are 1.6 (1.5) and 1.5 (1.6), respectively. The thicknesses of the positive (bottom) and negative (top) plates in mode 1 are 1.852 and 1.736 \( \mu m \) and are 1.852 \( \mu m \) in both plates of mode 2. These values were chosen to meet the ideal dark and bright state conditions in Sec. II under crossed polarizers. We use these parameters to calculate optical properties of our models in this article.

First, with the viewing direction in the director plane \([\text{Fig. } 5(a)]\) of our bright state model in mode 1, the projection of the optic axes of the positive A plate and negative C plate onto the plane perpendicular to the light propagation vector \((K)\) are parallel to each other. Consequently, the total effective retardation \((\Delta_{\text{white}})\) of the bright state is the summation of both values

\[
\Delta_{\text{white}} = \Delta n d_{\text{eff}}^{\text{positive}} + \Delta n d_{\text{eff}}^{\text{negative}}.
\]

Figure 6(a) shows how the effective retardation changes, as calculated from Eqs. (7), (8), and (10), in the director plane of our bright state model in mode 1 after considering the refraction of the incident light at the air interface. In the normal direction, the birefringence of the negative C plate is zero as expected, so it does not contribute to the total retardation. As the viewing angle increases from the normal direction, the effective retardation of the positive A plate and the negative C plate always decreases. The total effective retardation of both plates then decreases steeply as the viewing angle increases in the director plane.

Figure 6(b) is the corresponding figure for mode 2. It is almost the mirror image of Fig. 6(a). Intuitively, this feature of mirror symmetry is expected from the director structures in mode 1 and 2, i.e., positive A plate in mode 1 to negative A plate in mode 2 and negative C plate in mode 1 to positive C plate in mode 2. In this mode, unlike mode 1, the effective retardation of each layer (positive C plate and negative A plate) and their total increase as the viewing angles increase from the normal direction, but the sign of the total effective birefringence is negative. Consequently, the absolute value
of the total effective retardation decreases as the viewing angle increases, and this is the same trend as in mode 1.

Second, unlike in the director plane, for viewing angles out of the director plane [Fig. 5(b)] of our bright state model in mode 1, the projection of the optic axes of the positive A plate and negative C plate onto the plane perpendicular to the light propagation vector (K) are perpendicular to each other. Therefore, the total effective retardation \( \Delta_{\text{white}} \) of the bright state in this direction should be the difference between them

\[
\Delta_{\text{white}} = \Delta n_{\text{eff}}^\text{positive} - \Delta n_{\text{eff}}^\text{negative}.
\]

The retardation variations of the bright state model in mode 1 [Fig. 4(c)] out of the director plane are calculated from Eqs. (7), (8), and (11) and are shown in Fig. 7(a). The contribution of the negative C plate is exactly the same as when in the director plane. However, in this direction, the retardation of the positive A plate rises continuously as the viewing angle increases due to the increase in effective thickness. The total effective retardation of the bright state is the difference between the retardation values of the positive A plate and negative C plate, as mentioned earlier. Consequently, the value increases as the viewing angle increases from the normal direction.

Figure 7(b) shows the calculation results for mode 2 by using the same method as mode 1. This one is also a near-mirror image of the figure in mode 1 [Fig. 7(a)]. The total effective retardation decreases as the viewing angle increases and it has a negative sign. Therefore, the absolute value of the total effective retardation always increases as the viewing angle increases out of the director plane. We saw this same result in mode 1, so we can expect that the optical properties of the bright states are similar between mode 1 and mode 2, even though their director structures are different.

D. Analyses of the transmittance

From the retardation analyses, we know that although we use an ideal compensator for perfect optical compensa-
IV. CONCLUSIONS

We calculated the off-axis light transmission properties of the bright state of most common liquid crystal devices such as ECB, VA, TN, Pi cell, and symmetric splay cell whose dark states were optically compensated to have minimum transmittances for all viewing angles. From the results of these calculations, we found that their bright states have a universal viewing angle shape in spite of completely different director structures in their liquid crystal layers.

In order to understand this strange phenomenon, we made simple dark and bright state models describing general liquid crystal devices and analyzed them in terms of effective retardation and transmittance. In accordance with these analyses, the total effective retardation in the director plane constantly falls as the viewing angle increases (“bell shape”). On the contrary, the total effective retardation out of the director plane consistently rises in the same situation (“reversed bell shape”). These retardation changes cause the transmittance changes. In the director plane, the transmittance decreases as the viewing angle becomes larger because the birefringence decreases in that direction. On the other hand, the transmittance out of the director plane increases first and then falls after the specific viewing angle if the liquid crystal layer is optically designed so that the transmittance of the normal direction is lower than the maximum value.

These viewing angle features of our bright state models agree well with the properties of most common liquid crystal devices, not only in the two main viewing planes, but also for all viewing directions. Therefore, we can say that our simple model can reasonably describe the optical properties of the real liquid crystal devices considered here. Accordingly, our simple model can be used to analytically understand and predict the optical properties, such as transmittance, luminance distribution and color analyses of current LCDs or possible candidates of new display modes because usually analytical methods for optical calculations are almost impossible in real devices.

Based on these results, we can say that the single domain LCD modes, considered here, whose dark states are optically compensated to give minimum transmittance, inevitably have asymmetric shapes of the off-axis light transmission properties between the director and out of the director planes in their bright states. Therefore, in order to achieve isotropic shapes of the bright state viewing angle properties, multidomain liquid crystal modes are necessary.