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Liquid crystal optical phase plate with a variable in-plane gradient

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We propose a nematic liquid crystal (LC) optical phase plate, with a large continuous in-plane gradient that is variable, and its application to a beam steering device with high efficiency. The device is a vertically aligned, continuous phase, optical phased array (V-COPA) that uses a negative dielectric anisotropy LC material. High steering efficiency of over 95% is demonstrated by modeling the LC director field and its effect on transmitted light. The period of the V-COPA grating can be varied by adjusting an applied voltage profile, which allows for continuous angular control of the diffraction angle. 

I. INTRODUCTION

A nonmechanical optical beam steering device would be useful in many applications such as optical free space communications, infrared counter measures, projection displays, and optical fiber switches. Liquid crystal (LC) technology has become a promising and excellent candidate for nonmechanical optical beam steering components because of low driving voltage, large birefringence, and low cost fabrication techniques.

Continuously steerable, LC based devices follow the basic idea of generating a linear change of optical path difference (OPD) across the aperture, which tilts the incident phase front and thereby steers the optical beam. The steering angle is determined by the magnitude of the phase gradient caused by the spatially varying retardation \( \Delta n d \) (\( \Delta n \) is the effective birefringence and \( d \) is the LC cell thickness).

In a simple approach, \( \Delta n \) is varied to achieve a gradient, however to achieve a large gradient over a large aperture with a continuous phase profile requires a large value of \( d \). The large value of \( d \) leads to slow response, absorption, and scattering of light by the device. In 1990s, optical phased array (OPA) technology was introduced into beam steering by McManamon et al. The OPA approach fixes the problem of large \( d \), found in the simple approach by introducing “resets” into the phase profile whose OPD step is an integer number of wavelengths of the light being considered. These resets, if mathematically ideal, would not be a large problem for a single frequency waveform, however, in real devices it is well known that they degrade the achievable device efficiency. The OPA devices that were described by McManamon et al. had an efficiency of 85% for steering 1 \( \mu m \) radiation to 1.5°. Higher efficiencies were subsequently reported by Resler et al. for 10.6 \( \mu m \) radiation.

Another approach toward generating a phase gradient is based on Pancharatnam’s method that uses a stack of a quarter-wave plate, a half-wave plate, and another quarter-wave plate (a “QHQ” stack).

Figure 1 shows the optical model of the QHQ device of Pancharatnam: A light beam passes through a polarizer, a quarter-wave plate (\( \lambda/4 \) plate), a half-wave plate (\( \lambda/2 \) plate), another quarter-wave plate, and another polarizer.

The incident light becomes a circular polarized light after the first \( \lambda/4 \) plate so that the light incident on the half-wave plate can be defined as \( E_{in} \) according to Jones Calculus notation

\[
E_{in} = \begin{bmatrix} E_{x_{in}} \\
E_{y_{in}} \end{bmatrix} = \begin{bmatrix} E_{x_{in}} \\ i \cdot E_{y_{in}} \end{bmatrix}.
\]

For convenience, we assume right hand circular polarized light.

The transmitted light leaving the half-wave plate, \( E_{out} \), is defined as a linear mapping of the incident light \( E_{in} \) by a Jones matrix which represents the \( \lambda/2 \) plate,

\[
E_{out} = R \cdot P \cdot R^{-1} \cdot \begin{bmatrix} E_{x_{in}} \\
E_{y_{in}} \end{bmatrix} = \begin{bmatrix} \cos \beta - \sin \beta \\
\sin \beta \cos \beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\
0 & e^{i\phi} \end{bmatrix} \cdot \begin{bmatrix} \cos \beta & \sin \beta \\
-\sin \beta & \cos \beta \end{bmatrix} \cdot \begin{bmatrix} E_{x_{in}} \\ i \cdot E_{y_{in}} \end{bmatrix},
\]

where \( R \) and \( P \) are defined as a rotation matrix and Jones matrix in the principle axis frame of the \( \lambda/2 \) plate, respectively, \( \beta \) represents the angle between the slow axis of the \( \lambda/2 \) plate and the \( x \) axis, and \( \phi \) is denoted as the phase retardation of the \( \lambda/2 \) plate, which is equal to \( \pi \) here. The final relationship can be simplified as

![FIG. 1. Schematic of a QHQ stack. [(a) and (e)] Polarizer, [(b) and (d)] quarter-wave plate, and (c) half-wave plate.](image-url)
Based on an increasing OPD need resets that limit the efficiency, continuously variable, optical beam steering devices that are not continuously variable. The devices are switchable in different alignment techniques to generate a fixed azimuthal angle called a polarization grating. Several research groups have reported work based on this idea. For example, Honma and Nose\textsuperscript{10} showed a lens fabricated through a microrubbing technique. Crawford \textit{et al.} demonstrated a LC diffraction grating through a polarization holography exposure on a linear photopolymerizable polymer alignment layer.\textsuperscript{11} Escuti and Jones\textsuperscript{12} reported their LC polarization grating to modulate unpolarized or polarized light with a grating period as small as 6.3 μm. These results have demonstrated the validity of this basic idea of what is called a polarization grating.

Unfortunately, all the above QHQ methods have utilized different alignment techniques to generate a fixed azimuthally distributed LC director. The devices are switchable in that their steering effect can be turned on or off through the application of an electric field along the device normal, but are not continuously variable.

In summary, we have shown that previously demonstrated continuously variable optical beam steering devices based on an increasing OPD need resets that limit the efficiency of the device. We have also shown that high efficiency devices based on the Pancharatnam QHQ concept have been demonstrated, but they are not continuously variable.

It is the objective of this paper to demonstrate a high efficiency, continuously variable, optical beam steering device that is based on the Pancharatnam QHQ concept. We call our device a V-COPA device because it is based on the OPA idea, but has a continuous phase profile, and uses vertical alignment.

\section*{II. V-COPA DEVICE CONCEPT}

The V-COPA device uses a vertical alignment and a LC material that has a negative dielectric anisotropy. The basic idea is that the when a voltage profile is applied, the LC director will lie in the plane of the cell, but without a strong surface imposed in-plane direction. This allows for the pitch of the spiral (and the beam steering angle) to be changed. Two aspects of the design that need to be considered are as follows: how the initial spiral pattern is formed and how it can be changed.

One way we have considered to form an initial, defect-free, spiraling director configuration in the plane of the cell is to slightly modify the vertical alignment as will be explained here with reference to Figs. 3 and 4.
In Fig. 3, the alignment is approximately vertical at the top and bottom substrates. We call this a quasivertical alignment because the alignment is not exactly vertical in some domains, which are slightly tilted to the +y and –y directions (the director rotates about the x axis with alternating signs to tip in and out of the plane of the drawing). The alignment over electrode 3 and over the gap near it is slightly tilted to the +y direction. The alignment over electrode 6 and gap near it are slightly tilted to the –y. All of the other regions keep the initial vertical alignment.

With the above surface alignment, we consider that the rms average voltage applied to the entire cell is approximately the same, but a dc offset, that is a function distance along the x axis in Fig. 4, is employed to effect the spiral configuration. It is anticipated that at a location where the dc offset is changed to a higher value, the director will tend to orient in the x-z plane with the director rotated slightly from the horizontal with a particular rotational sense about the y axis. At another location where the dc offset is changed to a lower value, the director will tend to orient in the x-z plane with opposite rotational sense about the y axis from the horizontal. The intended effect of this will be to “trap” a pi rotation of the director between these two locations. The dc offset is changed through the use of patterned electrodes.

Combining the modified vertical alignment with the voltage offsets, the V-COPA device can achieve a defined, defect-free, initial spiral director structure. For example, referring to Fig. 3, the dc offset could be changed between electrodes 1 and 2, and between electrodes 4 and 5.

To vary the pitch of the spiral, the location the dc offset changes can be moved by changing the voltage applied to the electrodes. Because of the trapped pi rotation between their locations, if they are moved closer together, the pitch will be shortened.

As an example of a device built with the V-COPA concept, we consider a 5 μm (gap thickness d along the z axis) cell filled with negative dielectric anisotropy (Δe) LC (MLC-6608) with: Δe=-4.2, n_o=1.4748, n_e=1.5578, and elastic constants K_{11}=16.7 pN, K_{22}=7.0 pN, and K_{33}=18.1 pN. The LC cell, as shown in Fig. 4, consists of top and bottom substrates with patterned electrodes. The electrode width (w_x) along the x axis is 4 μm, the length (l_z) is 1 μm along the y axis, and the gap width (w_y) between each electrode is 1 μm.

To control the dc offset along the x direction while maintaining a constant rms voltage, for example, voltages in the bottom electrodes might be 10, 12, 12, 10, 10, 10, and 12 V, while 0, 2, 2, 0, 0, 0, and 2 V are applied on the top.

III. NUMERICAL SIMULATIONS OF THE V-COPA DEVICE

A two dimensional model, where the director orientation is considered to be a function of x and z, is used in our modeling. A generalized force on the director components (n_x, n_y, and n_z) may be written as

\[
[f_G]_{ni} = \frac{\partial \mathbf{f}_G}{\partial n_{i}} = \frac{\partial f_G}{\partial n_{i}} = \sum_{j=x,y,z} d\left[ \frac{\partial f_G}{\partial n_{j}} \right] i = x, y, z,
\]

where \( f_G \) is the Gibbs free energy density, formed from the Frank–Oseen strain free energy density and the electric free energy density, and \( -\frac{\partial f_G}{\partial n_{i}} \) is the functional Euler–Lagrange derivative of the free energy density.

The general update formula for each grid is the balance between the elastic torque and the viscous torque given in (Eq. (5))

\[
\gamma \frac{dn_{i}}{dt} = -\frac{\partial f_G}{\partial n_{i}} = \lambda n_{i}
\]

where \( \gamma \) is the viscosity coefficient and \( \lambda \) is a Lagrange multiplier used to maintain the unit length of the director \(|n| = 1\).

The derivatives in Eq. (5) are replaced by finite differences,

\[
\frac{\Delta n_{i}}{\Delta t} = -[f_G]_{ni},
\]

\[
\Delta n_{i} = -\frac{\Delta t}{\gamma}[f_G]_{ni},
\]

\[
n_{i}^{\text{new}} = n_{i}^{\text{old}} - \frac{\Delta t}{\gamma}[f_G]_{ni}.
\]

The director components are represented by \( n_x, n_y, \) and \( n_z \) at discrete points on a two dimensional spatial grid. During each update of the director components on each grid point, the time derivatives are taken in the forward direction and the spatial derivatives are computed with second order accurate central differences. The grid size used in our calculation was 140 × 40 and the grid spacing was 0.25 × 0.125 μm² for x and z dimensions, respectively. In our discretized numerical model, we enforced the unit length of the director by normalization rather than using the Lagrange multiplier method. The equilibrium state was assumed to be reached when the tolerance (which is the absolute value of the difference between the current iteration’s average \( ||f_G||^{\text{new}} \) and previous iteration’s average \( ||f_G||^{\text{old}} \) ) was less than 10^{-7} N/m².

After the director structure is obtained, the phase profile of output light is obtained by the Jones Calculus method, including the quarter-wave plates and polarizers shown in Fig. 1.

With the above phase profile of transmitted light in the near field, the diffraction efficiency can be calculated by Kirchhoff’s scalar diffraction theory, as shown in Eq. (7)

\[
E_f \propto \int_{S} E_{a} \left(1 + \cos \theta \right) \frac{e^{ikr}}{\sqrt{r}} d\mathbf{r},
\]

where \( E_a \) and \( E_f \) are the complex electric field amplitude of light in the near and far field, respectively, and \( \mathbf{r} \) represents the vector from source point in the near field to an objective.
point in the far field. \([1 + \cos \theta]/2\) is the obliquity factor where \(\theta\) is the angle between \(\mathbf{r}\) and objective surface normal. The graphs plotted, using this equation, of the far field intensity of light transmitted through a V-COPA device were normalized to the peak height calculated with no V-COPA device present. With this procedure the graphs shown can be related directly to the diffraction efficiency of the phase profile in the aperture.

**IV. RESULTS**

**A. Establishment of a spiral director configuration**

Using the modeling methods discussed in Sec. III and the simulation parameters of the device described in Sec. II we demonstrate the formation of a defect-free spiral structure based on our concept. Initially the director field is determined by the surface alignment condition with no electric fields applied (nearly homeotropic). Then voltages are applied to the eight electrodes (bottom electrodes: 10, 12, 12, 10, 10, 10, and 12 V; top electrodes: 0, 2, 2, 2, 0, 0, 0, and 2 V). After LC directors relax to the equilibrium state by the vector field method, we get the final director configurations, as shown in Fig. 5 where a nearly perfect spiral director configuration is formed. The arrows represent the director orientation, and the curves are the equipotential lines. We can define the period of the spiral configuration as the distance between the regions where the director is substantially along the \(y\) axis in the figure. We will refer to those locations on the \(x\) axis as \(x_{V+}\) (where the director is along the \(+y\) axis) and \(x_{V-}\) (where the director is along the \(-y\) axis). Note that these locations are halfway between the locations \(x_{V+}\) and \(x_{V-}\) in Fig. 5 (the \(x\) values where the potential steps up and down, respectively). Please note the twist sense of the helix (as observed in the top view of Fig. 5) would be opposite if the voltages on the bottom electrodes were exchanged (10 V exchanged with 12 V), and if the voltages on the top electrodes were also exchanged. To make this clearer, if the order of the \(x\) locations is \(x_{V+}>x_{V+}>x_{V+}>x_{V-}\), then the helix will have a particular rotational sense; but if it is \(x_{V+}>x_{V-}>x_{V-}>x_{V+}\), then it will have the opposite rotational sense.

**B. Tuning the period of the spiral grating**

After the initial spiral director configuration is achieved, by translating the regions where the director is trapped in the \(x-z\) plane, we can expand or shrink the period of the grating. This is accomplished by moving the locations of the steps in the potential \((x_{V+} \text{ and } x_{V-})\). For this demonstration, instead of using periodical boundary condition, a “stress-free” boundary condition is used at the \(x\) locations of the left and right sides of the computed area shown in Figs. 6 and 7. The director orientation at these boundaries is determined by a linear extrapolation from the three grid points closest to the boundary along the \(x\) direction.

To show the ability to modify the spiral pattern we start with the director configuration, Fig. 5 and apply a new voltage profiles to the LC cell, that are 10, 12, 12, 12, 10, 12, and 12 V on bottom electrodes and 0, 2, 2, 2, 2, 0, 2, and 2V on top ones. Loading the previous director structure in Fig. 5 as the initial starting configuration, the director will reach a new equilibrium state. Figure 6 shows the new director configuration and the equipotential lines. It can be seen that due to the change in the applied potential, the \(x_{V-}\) and \(x_{V+}\) regions on the right side of the figure are moved toward each other, while the \(x_{V+}\) region on the left side is held in position. By then changing the voltage profile on bottom to...
10, 10, 12, 12, 10, 12, and 12 V, then 10, 10, 10, 12, 12, 10, 12, and 12 V, and then to 10, 10, 10, 10, 12, 10, 12, and 12 V (with corresponding changes on the top electrodes to maintain the rms voltages), we can move the \( xV^+ \) region on the left side of Fig. 6 toward the right, while holding the position of the other \( xV^+ \) and \( xV^- \) regions in Fig. 6. The final resulting potential and director configuration is shown in Fig. 7.

While Figs. 6 and 7 demonstrate the concept of controlling the director configuration in this device, in many applications we are interested in changing the period of a spiral-configuration. To demonstrate this, we can start with the director and potentials shown in Fig. 5, move the periodic boundary conditions to the new pitch length, and then adjust voltages on the electrodes to provide the new director profile. Figure 8 shows the result of this procedure, where the left and right sides of this figure correspond to the new locations of the periodic boundary conditions. The location of the new boundaries were placed in the director profile of Fig. 5 at the \( x \) values of 9.75 and 29.75, and the voltages on the electrodes in the included region were changed to 10, 12, 10, and 10 V at the bottom and 0, 2, 2, 0, and 0 V on top. The new spiral period is 10.25 \( \mu \text{m} \).

C. Optical efficiency

Using the modeling methods and cell parameters previously described, we calculate the far field intensity pattern for the case of calculated periodic spiral director fields with the periods shown in Sec. IV B. In these calculations a linear polarized beam (632.8 nm) passes through the QHQ system in Fig. 1 except that the \( \lambda/2 \) plate is replaced by the V-COPA LC cell. A Gaussian profile beam is used, as shown in Fig. 9(a).

Figure 9(b) is calculated for the spiral period of 15.75 \( \mu \text{m} \) shown in Fig. 5 and shows a deflection angle of 2.302° and a steering efficiency of 98.27%. In Fig. 9(c), the transmitted light across a smaller spiral period of (10.25 \( \mu \text{m} \)) from Fig. 8 is also calculated, showing a steering angle of 3.54° with an efficiency of 96.77%. Once the desired director configuration is established it will likely be possible to apply a uniform voltage to the top and bottom electrodes (for example, 10 V on all of the top electrodes and 0 at the bottom). When doing this, we find that the efficiency climbs very close to 100%.

V. DISCUSSION

A key aspect of the V-COPA device is the trapping of a pi-twist region of the in-plane director configuration through the use of voltage offsets.

As may be clear from inspecting Figs. 5–7, this is accomplished, in part, by fixing the director in the \( x-z \) plane at the locations \( xV^+ \) and \( xV^- \). A director fixed in this way traps a pi twist of the director in the space between them. The sense of this twist is fixed by the initial spiral sense set by the spatially patterned quasivertical surface alignment and the dc offsets.

The reason why the director at the regions \( xV^+ \) and \( xV^- \) near the surfaces is oriented, as shown, is primarily due to the torques imposed on it by the electric field and the vertical surface alignment layer. Because the LC material used has a negative dielectric anisotropy, the director (especially in the high field region near the surface) will tend to align perpendicular to the electric field direction. This effect coupled with the effect of the alignment layer to tilt the director perpendicular to the surface causes the director to tend to align perpendicular to the axis of the electrodes and along the equipotential lines.
The above results show that a device built, as described, can provide exceptional performance. However modifications of this example device can be considered. For example, two-frequency LC materials could be used with a low frequency potential generated by voltage differences between electrodes on the same surface and with a high frequency potential generated by voltage differences between electrodes on opposite surfaces. Also, flexoelectric or smectic \( C \) materials can be considered for the used LC. Other types of optical devices, such as lenses, can be made by causing the spiral to have a spatially dependent period.

**VI. CONCLUSION**

A variable beam steering device with an in-plane spiraling optic axis has been proposed. Employing a LC material with a negative dielectric anisotropy, quasivertical surface
alignment, and an electric field with an offset voltage profile, excellent spiral configurations have been demonstrated. The resulting structure shows very high diffraction efficiency (95%–98%). Compared to conventional LC prisms and OPA devices, the V-COPA device inherently has a continuous phase profile with no resets, while its thin (half-wave retardation) cell thickness yields high speed, low absorption, and low scattering. Moreover, the angle tunability of this device based on Pancharatnam’s idea is achieved. Other optical devices such as electrically controlled lenses can also be envisioned based on this approach.

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